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NZQA

Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Level 3 Mathematics and Statistics (Statistics) 2023

91586 Apply probability distributions in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability distributions in solving problems.	Apply probability distributions, using relational thinking, in solving problems.	Apply probability distributions, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (DO NOT WRITE IN THIS AREA). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

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The assessment starts on the following page.

QUESTION ONE

(a) There are many sheep farms in New Zealand, where animals graze on pasture all year round. The stocking rate is defined as the number of animals grazing on a given amount of land. For sheep being farmed on New Zealand hills and mountains, average stocking rates are **9 sheep per hectare**.

(i) Use a suitable probability distribution to estimate the probability that there are more than 10 sheep on a given hectare of New Zealand hills and mountains.

State the name and parameter(s) for your chosen probability distribution as part of your answer.

(ii) It is commonly known that sheep tend to flock together (form groups).

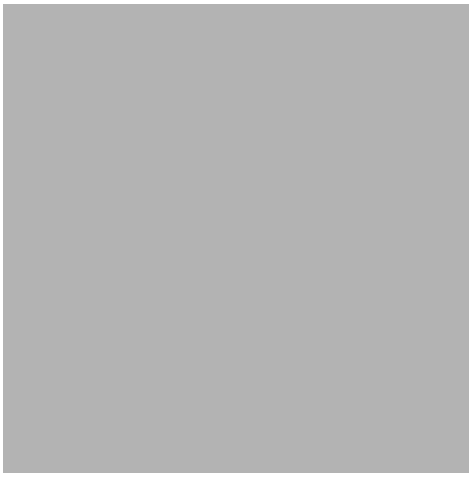
Give TWO reasons why this piece of information may affect your choice of probability distribution and its required conditions for modelling the number of sheep per hectare on New Zealand hills and mountains.

Reason one:

Reason two:

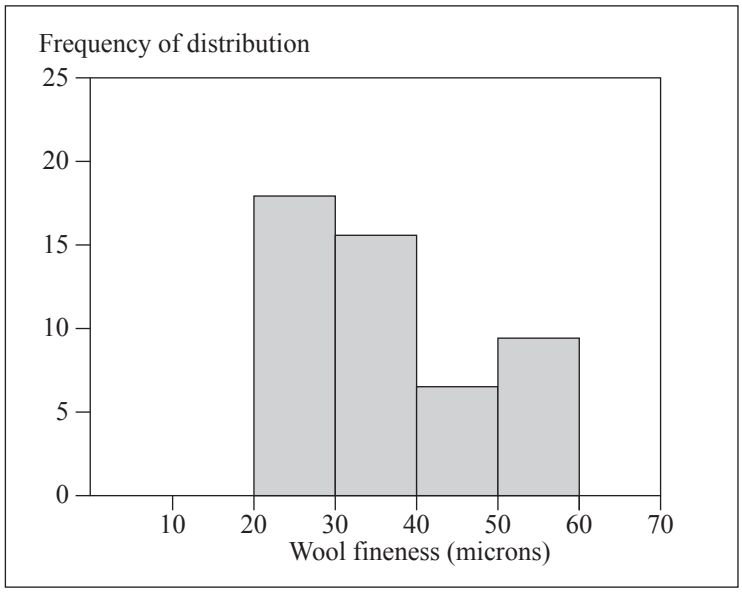
(b) Awassi sheep are a rare New Zealand sheep breed with origins in the Middle East. They are predominantly milking sheep, although they also produce wool that is suited to the carpet industry.

Wool fineness is a measure of the diameter of the sheep wool fibres (in microns). Figure 1(a) shows the frequency distribution of wool fineness for a sample of 49 Awassi sheep.



Source: https://nzsheep.co.nz/awassi/

Figure 1(a): Frequency histogram of wool fineness in Awassi wool (n = 49)



Source: Lewin, Horowitz & Zacks, 1957

(i) Suppose a uniform distribution is chosen to model the wool fineness of Awassi sheep.

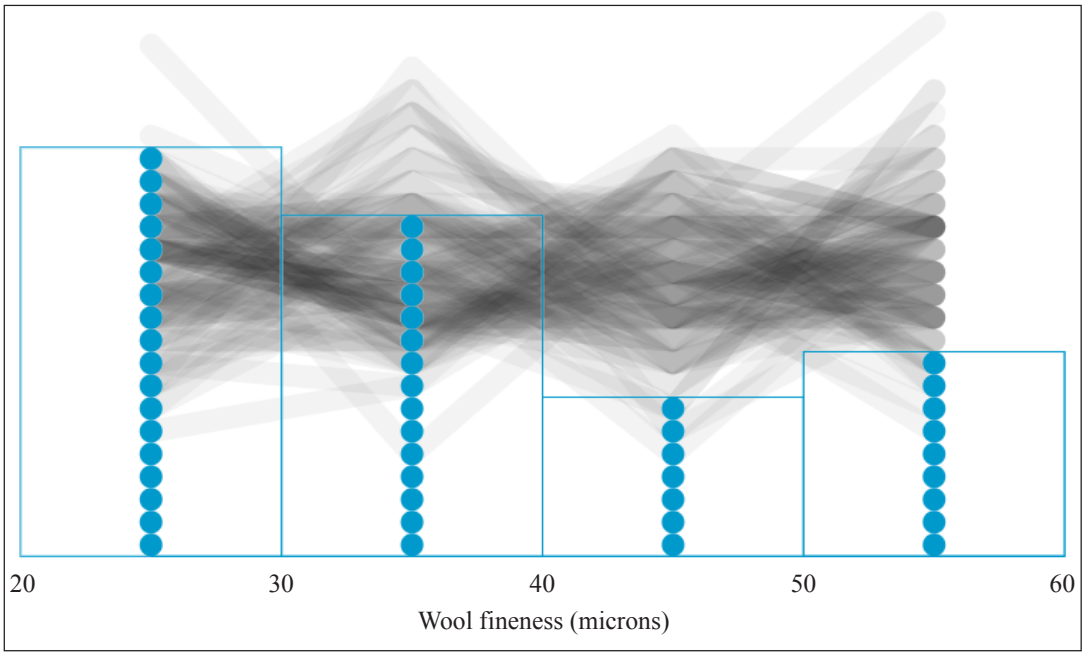
Use this probability distribution to calculate the probability that for a randomly selected Awassi sheep, the wool fineness is greater than 35 microns.

As part of your answer, state the parameters used.

(ii) The sample data shown in Figure 1(a) is run through a simulation model 1000 times, assuming that the wool fineness for Awassi sheep is uniformly distributed, with a minimum of 20 microns and a maximum of 60 microns.

Figure 1(b) shows the results of the simulation model (overfitted-shape for the model generated data) and original real observed data. The blue dots show the relative frequencies of the different classes of wool fineness. The grey band shows the variation expected for classes of wool fineness, based on a total of 49 sheep and ¼ probability for each wool class.

Figure 1(b): Results of the simulation model and original real data



Based on the results of the simulation model and the original real observed data (Figure 1(b)), discuss whether the uniform distribution model presented above appears to be appropriate for modelling the wool fineness for Awassi sheep.

Support your answer with statistical reasoning.

(iii) Using an appropriate probability distribution model, calculate an estimate for the probability that the wool fineness is between 25 and 40 microns for each of four Awassi sheep.

State the name and parameters for your chosen probability distribution as part of your answer.

(iv) One of the assumptions needed for the calculation in part (iii) is independence of events.

Identify the appropriate event and justify this assumption in context.

QUESTION TWO

https://www.thestatesman.com/supplements/8thday/quality-egg-issue-1502756802.html

(a) The rate at which a hen lays eggs can be modelled by a Poisson distribution, and 99.73% of the time she lays at least one egg each week.

(i) Calculate an estimate for the average number of eggs the hen lays each week.

(ii) What is the most likely number of eggs that the hen will lay each week?

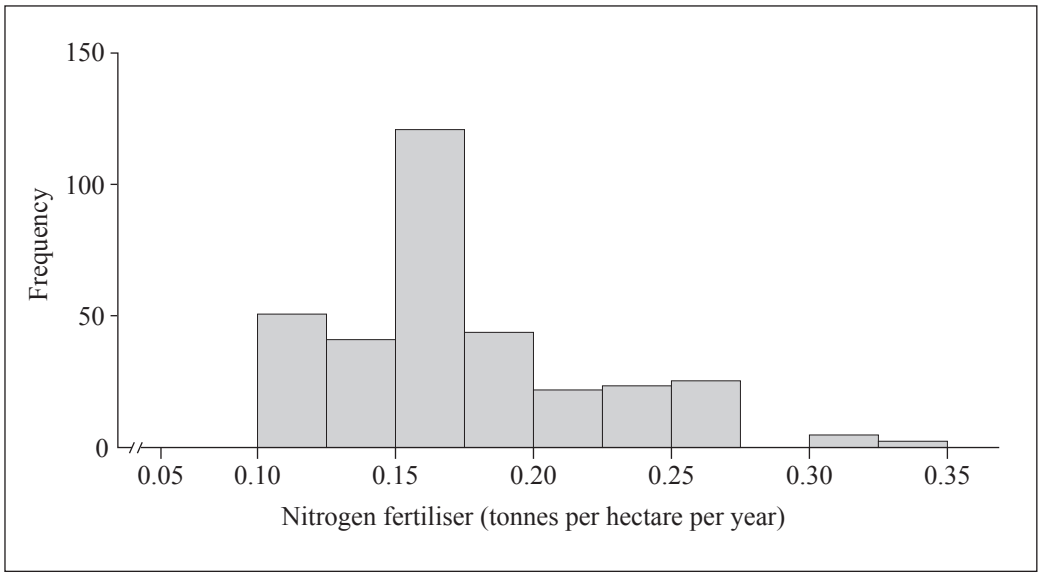
Justify your answer with statistical reasoning or calculations.

(iii) Over a period of time, the hen lays 10 eggs. Each egg independently does or doesn't hatch, with probability 0.65 of hatching.

Calculate the probability that 9 or more of the 10 eggs will hatch.

(b) The following figure shows the distribution of the amount of nitrogen fertiliser (tonnes) applied per hectare per year on 332 New Zealand dairy farms.

Figure 2: Nitrogen fertiliser (tonnes) applied per hectare per year on New Zealand dairy farms



Source (adapted): Doole, G. J., Marsh, D., and Ramilan, T. (2013). Evaluation of agri-environmental policies for reducing nitrate pollution from New Zealand dairy farms accounting for firm heterogeneity. *Land Use Policy*, 30 (1), 57–66. 10.1016/j.landusepol.2012.02.007.

(i) Using the data in Figure 2, suggest a suitable probability distribution for modelling the amount of fertiliser used per hectare per year on New Zealand dairy farms.

Assume that the most common amount of nitrate fertiliser applied is 0.16 tonnes per hectare per year.

State the name and parameter(s) for your chosen probability distribution AND justify your choice of distribution and parameter(s) as part of your answer.

Name and parameters of distribution: _____

Justification: _____

(ii) Ten New Zealand dairy farms are chosen at random.

Use the probability distribution you proposed in part (i) to estimate the probability that more than half of these 10 farms applied less than 0.16 tonnes of nitrogen fertiliser per hectare per year.

QUESTION THREE

(a) Individual cows produce different total amounts of milk over the milking season. For New Zealand dairy cows, suppose the total kilograms of milk per season per cow can be modelled by a normal distribution, with a mean of 4370 kg and standard deviation of 1350 kg.

https://agfundernews.com/new-zealand-dairy-appeals-to-pension-funds-as-firm-launches-second-fund-despite-milk-price-lows-exclusive010

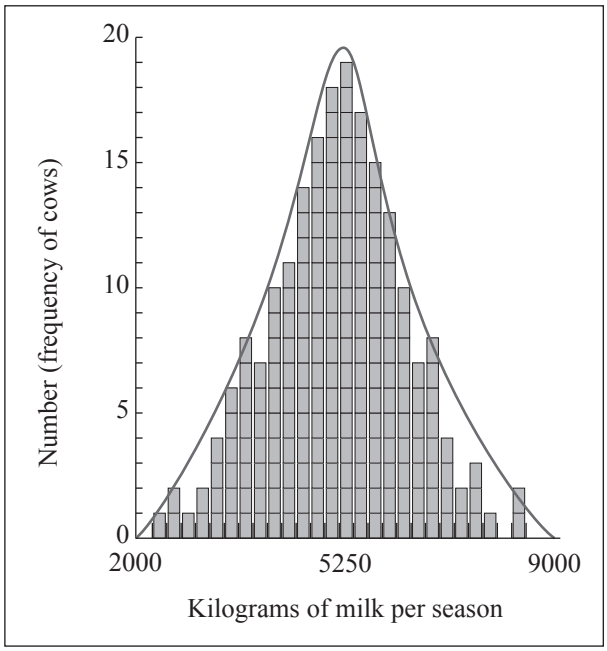
(i) It is known that for **South Island** dairy cows, approximately 30% of the cows produce less than 4290 kg of milk per season per cow.

Discuss whether the normal distribution model described above appears to be appropriate for modelling milk production per season per cow for South Island dairy cows.

Support your answer with statistical calculations.

(ii) Milk production data is collected from a herd of 200 dairy cows (Figure 3 below).

Figure 3: Kilograms of milk per season per cow for 200 dairy cows



Source (adapted):/ /bizplan-uz.com/learning/course/?COURSE_ID=6&LESSON_ID=439&LESSON_PATH=8.436.438.439

For this herd, 10% of dairy cows have milk production of less than 3600 kg per season.

Suggest suitable values of the parameters of a normal distribution model that could be used for modelling milk production per season for this particular herd.

Support your answer with statistical calculations and reasoning.

(iii) Compare the normal distribution models for the total kilograms of milk per season per cow produced by the 200 cows in part (ii), and the total kilograms of milk per season per cow produced by all New Zealand dairy cows.

Make at least TWO comments and support your discussion with calculations and sketches where appropriate.

(b) The table below shows the probability distribution of the random variable N, the number of goat kids (baby goats) born per pregnancy for this type of dairy goat.

n	0	1	2	3	4
$P(N = n)$	0.1	0.12	0.75	0.02	0.01

https://www.nzdgba.co.nz/

(i) Calculate the mean and standard deviation of the number of goat kids born per pregnancy for this breed of goat.

Mean: _____

Standard deviation: _____

(ii) Dairy goat farmers prefer types of dairy goat that produce an average of two goat kids per pregnancy.

Comment on whether or not this data suggests that this type of dairy goat will be a good choice for dairy goat farmers.

Question Three continues on the next page.

(iii) Suppose that, with improved nutrition of goats throughout the year, the proportion of pregnancies that result in two or more goat kids changes.

The table below shows the probability distribution of the random variable N, the number of goat kids born per pregnancy for this type of dairy goat with improved nutrition of goats throughout the year.

n	0	1	2	3	4
$P(N = n)$	0.03	0.07	0.53	0.25	0.12

Without further calculations, use statistical reasoning to explain what effect this change in probabilities would have on the expected value and standard deviation of the number of goat kids per pregnancy for the dairy goats.

Effect on expected value:

Effect on standard deviation:

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER