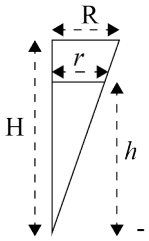
Assessment Schedule – 2013

Calculus: Apply integration methods in solving problems (91579)

Evidence Statement

ONE	Expected Coverage	Achievement	Merit	Excellence
(a)	$-2x^{-1}+\frac{x^{-2}}{2}+c$	Correct answer.		
(b)	$\pi x-\frac{1}{2}e^{2x}+c$	Correct answer.		
(c)	$v=6-\frac{5}{t+1}$ $d=\int v\,dt=\int\left(6-\frac{5}{t+1}\right)dt$ $=6t-5\ln(t+1)$ $t=4\quad d=24-5\ln 5=15.95$ $t=3\quad d=18-5\ln 4=11.07$ Distance travelled = 4.88 m	Correct integration.	Correct answer.	
(d)(i)	$\frac{dV}{dt}=kV$			
(ii)	$\int\frac{dv}{V}=\int k\,dt$ $\ln aV=kt$ $V=\frac{1}{a}e^{kt}$ $=Ae^{kt}$ $t=0$ months in May 2007 So $t=60$ months in May 2012 And $t=78$ months in Nov 2013 $365000=Ae^{k(60)}$ $A=\frac{365000}{e^{60k}}$ $382000=\frac{365000}{e^{60k}}\times e^{78k}$ $\ln\left(\frac{382000}{365000}\right)=\ln(e^{18k})$ $18k=0.045523$ $k=0.002529$ $A=\frac{365000}{e^{0.002529t}}$ = \$313 611 when $t=0$	Correct DE and integration.	Correct DE, integration and general solution to DE with logical working.	Correct answer including correct DE, integration and general solution to DE with logical working. Units not required in answer.

(e)	**Cylindrical tank** $E = \int_0^{\mathrm{H}} \mathrm{k}(\mathrm{H}-h)\,A(h)\,\mathrm{d}h$ $= \int_0^{\mathrm{H}} \mathrm{k}\pi\mathrm{R}^2(\mathrm{H}-h)\,\mathrm{d}h$ $= \mathrm{k}\pi\mathrm{R}^2\left[\mathrm{H}h - \frac{h^2}{2}\right]_0^{\mathrm{H}}$ $= \frac{\mathrm{k}\pi\mathrm{R}^2\mathrm{H}^2}{2}$ **Conical tank**  $A = \pi\left(\frac{\mathrm{R}h}{\mathrm{H}}\right)^2$ $E = \int_0^{\mathrm{H}} \mathrm{k}(\mathrm{H}-h)\,A(h)\,\mathrm{d}h$ $= \int_0^{\mathrm{H}} \mathrm{k}(\mathrm{H}-h)\pi\frac{\mathrm{R}^2h^2}{\mathrm{H}^2}\mathrm{d}h$ $= \frac{\mathrm{k}\pi\mathrm{R}^2}{\mathrm{H}^2}\int_0^{\mathrm{H}} (\mathrm{H}h^2 - h^3)\,\mathrm{d}h$ $= \frac{\mathrm{k}\pi\mathrm{R}^2}{\mathrm{H}^2}\left[\frac{\mathrm{H}h^3}{3} - \frac{h^4}{4}\right]_0^{\mathrm{H}}\mathrm{d}h$ $= \frac{\mathrm{k}\pi\mathrm{R}^2}{\mathrm{H}^2}\left[\frac{\mathrm{H}^4}{3} - \frac{\mathrm{H}^4}{4}\right]$ $= \frac{\mathrm{k}\pi\mathrm{R}^2\mathrm{H}^2}{12}$ $\therefore$ Energy for cone is $\frac{1}{6}$ energy of cylinder since $\frac{\mathrm{k}\pi\mathrm{R}^2\mathrm{H}^2}{12} = \frac{1}{6}\left[\frac{\mathrm{k}\pi\mathrm{R}^2\mathrm{H}^2}{2}\right]$	One correct integration.	Correct energy for conical tank.	Both integrals correct and correct solution.

NØ = No response / no relevant evidence
N1 = ONE answer demonstrating limited knowledge of integration techniques
N2 = ONE correct integration
A3 = TWO of Achievement (u)
A4 = THREE of Achievement (u)
M5 = ONE of Merit (r)
M6 = TWO of Merit (r)
E7 = Excellence with minor errors ignored (t)
E8 = Excellence correct (t)

TWO	Expected Coverage	Achievement	Merit	Excellence
(a)	$\int_1^2 f(x)\,dx = \frac{1}{2}\times 0.2(1.41+2.24$ $+2(1.56+1.72+1.89+2.06))$ $=1.81$	Correct answer.		
(b)	$=\int_1^k 3(x)^{\frac{1}{2}}\,dx$ $=\left[2x^{\frac{3}{2}}\right]_1^k$ $=2k^{\frac{3}{2}}-2$	Correct answer.		
(c)	$y=\frac{1}{3}x^2=2x$ $x^2-6x=0$ $x=0$ or 6 $\text{Area}=\int_0^6\left(2x-\frac{1}{3}x^2\right)dx$ $=\left[x^2-\frac{1}{9}x^3\right]_0^6$ $=12$	Correct integral (or difference of two correct integrals).	Correct solution.	
(d)	$\frac{dV}{dt}=\frac{400}{(t+2)^2}=400(t+2)^{-2}$ $V=\int 400(t+2)^{-2}dt$ $=\frac{-400}{(t+2)}+c$ $t=0,\ \ V=20\Rightarrow 20=\frac{-400}{2}+c$ $c=220$ $At\ t=6,\ \ V=\frac{-400}{8}+220=170\text{ L}$	Correct integration.	Correct solution.	

(e)	$f'(x) = 1 - \frac{1}{3}x^2$ $f(x) = x - \frac{x^3}{9} + c$ $x = 0,\ y = 0, \therefore c = 0$ At $x = -1,\ f'(x) = \frac{2}{3}$ and $f(x) = \frac{-8}{9}$ Tangent has equation $y = \frac{2}{3}x - \frac{2}{9}$ Intersection at $x - \frac{x^3}{9} = \frac{2}{3}x - \frac{2}{9}$ $x = -1, 2$ Area between curve and tangent $= \int_{-1}^{2}\left(x - \frac{x^3}{9} - \left(\frac{2}{3}x - \frac{2}{9}\right)\right)\mathrm{d}x$ $= \frac{1}{9}\left[\frac{3x^2}{2} - \frac{x^4}{4} + 2x\right]_{-1}^{2}$ $= \frac{3}{4}$	Correct equation of tangent.	Correct expression for area required (could be difference of two integrals).	Correct solution.

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THREE	Expected Coverage	Achievement	Merit	Excellence
(a)	$-\frac{1}{2}\operatorname{cosec} 2x + c$	Correct integration.		
(b)	$\text{Area} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin 2x \, \mathrm{d}x$ $= \left[-\frac{1}{2}\cos 2x \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ $= \frac{1}{2}$	Correct integration.	Correct answer.	
(c)	$\int_{\mathrm{m}}^{2\mathrm{m}} \frac{2x+5}{x^2+5x} = \left[\ln\left(x^2+5x\right) \right]_{\mathrm{m}}^{2\mathrm{m}} = \ln 3$ $\left[\ln\left(4\mathrm{m}^2+10\mathrm{m}\right) - \ln\left(\mathrm{m}^2+5\mathrm{m}\right) \right] = \ln 3$ $\ln \frac{4\mathrm{m}^2+10\mathrm{m}}{\mathrm{m}^2+5\mathrm{m}} = \ln 3$ $\mathrm{m}^2 - 5\mathrm{m} = 0$ $\mathrm{m} = 0, 5$	Correct integration.	Correct answer.	
(d)	$\int \frac{\mathrm{d}v}{v^2} = \int -\mathrm{k}\,\mathrm{d}t$ $-\frac{1}{v} = -\mathrm{k}t + \mathrm{c}$ When $t = 0, v = u$ $\mathrm{c} = \frac{-1}{u}$ $\frac{-1}{v} = -\mathrm{k}t - \frac{1}{u}$ When $t = 1, v = \frac{u}{1+\mathrm{k}u}$	Correct integration.	Correct solution.	
(e)(i)	$\frac{\mathrm{d}T}{\mathrm{d}t} = \mathrm{k}(T - 18)$			
(ii)	$\int \frac{\mathrm{d}T}{T-18} = \int \mathrm{k}\,\mathrm{d}t$ $\ln\lvert T-18 \rvert = \mathrm{k}t + c$ At $t = 0, \ln\lvert 65-18 \rvert = c = \ln 47$ At $t = 2, \ln\lvert 62-18 \rvert = 2\mathrm{k} + \ln 47$ $\mathrm{k} = -0.03297$ $\ln\lvert 54-18 \rvert = -0.03297 \times t + \ln 47$ $t = 8.08$ minutes	Correct DE and integration.	Correct DE, integration and general solution to DE, with logical working and correct value of k.	Correct answer including correct DE, integration and general solution to DE with logical working. Units not required in answer.

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Marking codes

Codes that may have been used in marking this examination paper have meaning as follows.

#	Hash - where a candidate obtains a correct answer but continues with further, unnecessary, material that is incorrect but does not show that the candidate shows a lack of understanding or a contradiction.
C	Consistency - where a candidate has obtained an incorrect value within a question and subsequently used that value.
NC	Non-consistency - where a candidate has obtained an incorrect value or expression within a question and has not used that value or expression where it was subsequently required.
RAWW	Right answer, wrong working - where a candidate presents a correct answer but the working or reasoning leading to it is incorrect, incomplete or contains one or more errors.
R	Rounding error - where a candidate produces a correct sequence of calculations, but the answer does not agree to 2 significant figures with the answer given in the assessment schedule as a result of rounding a number in the sequence of calculations.
MEI	Minor error ignored - where a candidate has made a minor error and this has been ignored.
Two ans	Two answers given - where a candidate writes two answers, one correct and the other incorrect, and neither has been deleted (the correct answer is not accepted).

Judgement Statement

	Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
Score range	0 – 6	7 – 12	13 – 20	21 – 24