Assessment Schedule – 2014

Calculus: Apply integration methods in solving problems (91579)

Evidence Statement

Q1	Expected Coverage	Achievement u	Merit r	Excellence t
(a)	$2\ln x + 3x^{-1} + c$	Correct integral.		
(b)	$\left[3\tan x\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$ $= 3\tan\frac{\pi}{4} - 3\tan\frac{\pi}{6}$ $= 3 - \sqrt{3}$	Correct solution with correct integral. Accept 1.27 $3 - \frac{3}{\sqrt{3}}$		
(c)	$v(t) = 5\left(4 - 3e^{-0.2t}\right)$ $d = \int_0^{10} 5\left(4 - 3e^{-0.2t}\right)\mathrm{d}t$ $= 5\left[4t + 15e^{-0.2t}\right]_0^{10}$ $= 5\left[\left(40 + 15e^{-2}\right) - \left(0 + 15\right)\right]$ $= 5 \times 27.03$ $= 135.15\text{ m}$	Correct integral.	Correct integral and correct solution. Units not required.	
(d)	$\frac{dV}{dt} = k\sqrt{V}$ $\int V^{\frac{-1}{2}}\,dV = \int k\,\mathrm{d}t$ $2V^{\frac{1}{2}} = kt + c$ $t = 0 \Rightarrow 2\sqrt{2500} = 0 + c$ $c = 100$ $t = 2 \Rightarrow V = 2025$ $2\sqrt{2025} = 2k + 100$ $90 = 2k + 100$ $k = -5$ $\therefore 2\sqrt{V} = -5t + 100$ $V = 0 \Rightarrow 0 = -5t + 100$ $t = 20\text{ (days)}$	Correct integral (3rd line).	Correct solution with correct Integral. Units not required.	

(e)	$y = px^2 \Rightarrow y^2 = p^2x^4$ At intersection: $p^2x^4 = px$ $px(px^3 - 1) = 0$ $x = 0$ or $\frac{1}{\sqrt[3]{p}}$ Area $= \int_0^{\frac{1}{\sqrt[3]{p}}} \left(\sqrt{p}\sqrt{x} - px^2\right)\mathrm{d}x$ $= \left[\frac{2\sqrt{p}}{3}x^{\frac{3}{2}} - \frac{px^3}{3}\right]_0^{\frac{1}{\sqrt[3]{p}}}$ $= \frac{2p^{\frac{1}{2}}\left(p^{\frac{-1}{3}}\right)^{\frac{3}{2}}}{3} - \frac{p.\left(p^{\frac{-1}{3}}\right)^3}{3}$ $= \frac{2p^{\frac{1}{2}}p^{\frac{-1}{2}}}{3} - \frac{p.p^{-1}}{3}$ $= \frac{1}{3}$		Correct integral.	Correct solution with correct integral.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of integration techniques	ONE correct integration	2u	3u	1r	2r	1t with minor errors ignored	1t

Q2	Expected Coverage	Achievement	Merit	Excellence
(a)	$\sec x + \frac{\cos 2x}{2} + c$	Correct integral.		
(b)	$\frac{dy}{dx} = 2x^3 - 3x^2 + c$ $x = 2,\ \frac{dy}{dx} = 8 \Rightarrow 8 = 2\times 2^3 - 3\times 2^2 + c$ $8 = 16 - 12 + c \quad c = 4$ $\therefore \frac{dy}{dx} = 2x^3 - 3x^2 + 4$ $y = \frac{x^4}{2} - x^3 + 4x + k$ $y(2) = 10 \Rightarrow 10 = 8 - 8 + 8 + k$ $k = 2$ $\therefore y = \frac{x^4}{2} - x^3 + 4x + 2$	Correct solution with correct integrals given.		
(c)	Total area = $\int_0^{\pi} \sin\frac{x}{2}dx$ $= \left[-2\cos\frac{x}{2}\right]_0^{\pi}$ $= 2$ Want $\left[-2\cos\frac{x}{2}\right]_0^{k} = 1$ $-2\left(\cos\frac{k}{2} - 1\right) = 1$ $-2\cos\frac{k}{2} + 2 = 1$ $\cos\frac{k}{2} = \frac{1}{2}$ $\frac{k}{2} = \frac{\pi}{3}$ $k = \frac{2\pi}{3}$ OR $\int_0^{k} \sin\frac{1}{2}x\,dx = \int_k^{\pi} \sin\frac{1}{2}x\,dx$ $\left[-2\cos\frac{1}{2}x\right]_0^{k} = \left[-2\cos\frac{1}{2}x\right]_k^{\pi}$ $= -2\cos\frac{k}{2} + 2 = 2\cos\frac{k}{2}$ $\cos\frac{k}{2} = \frac{1}{2}$ $\frac{k}{2} = \frac{\pi}{3}$ $k = \frac{2\pi}{3}$	Correct total area with correct integral. OR Correct 3rd line in alternative method.	Correct solution with correct total area and correct integral.	

(d)	$\frac{dy}{dx}=\frac{3\sqrt{x}}{2y}$ $\int 2y\,dy=\int 3\sqrt{x}\,dx$ $y^2=2x^{\frac{3}{2}}+c$ $x=4,\ y=5\Rightarrow 25=2\times 8+c$ $c=9$ $\therefore y^2=2x^{\frac{3}{2}}+9$ $x=9:\quad y^2=2\times 9^{\frac{3}{2}}+9$ $=54+9$ $y=\sqrt{63}$	Correct general solution.	Correct solution.	
(e)	$\text{Area}=\int_0^4\left(x^{\frac{1}{2}}+1\right)dx$ $=\left[\frac{2x^{\frac{3}{2}}}{3}+x\right]_0^4=\frac{28}{3}$ $\bar{x}=\frac{3}{28}\int_0^4\left(x^{\frac{3}{2}}+x\right)dx$ $=\frac{3}{28}\left[\frac{2x^{\frac{5}{2}}}{5}+\frac{x^2}{2}\right]_0^4$ $=\frac{3}{28}\left[\frac{64}{5}+8\right]=\frac{78}{35}=2.229$ $\bar{y}=\frac{3}{28}\int_0^4\frac{\left(x+2\sqrt{x}+1\right)}{2}dx$ $=\frac{3}{56}\left[\frac{x^2}{2}+\frac{4x^{\frac{3}{2}}}{3}+x\right]_0^4$ $=\frac{3}{56}\left[\frac{272}{12}\right]=\frac{17}{14}=1.214$ $(\bar{x},\bar{y})=\left(\frac{78}{35},\frac{17}{14}\right)=(2.229,1.214)$	Correct area with correct integration.	Correct area with correct integration and 1 correct coordinate.	Correct area with correct integration and 2 correct coordinates.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of integration techniques	ONE correct integration	2u	3u	1r	2r	1t with minor errors ignored	1t

Q3	Expected Coverage	Achievement	Merit	Excellence
(a)	5	Correct solution.		
(b)	$\frac{3}{4}x^{\frac{4}{3}} + 2e^{3x-5} + c$	Correct solution.		
(c)(i) (ii)	$\frac{\mathrm{d}T}{\mathrm{d}t} = k(T-20)$ $\int \frac{dT}{T-20} = k\,\mathrm{d}t$ $\ln(T-20) = kt + c$ $T = A\mathrm{e}^{kt} + 20$ $t=0,\ T=220 \Rightarrow A = 200$ $t=3,\ T=100 \Rightarrow 100 = 200\mathrm{e}^{3k} + 20$ $0.4 = \mathrm{e}^{3k}$ $k = \frac{\ln 0.4}{3} = -0.3054$ $\therefore T = 200\mathrm{e}^{-0.3054t} + 20$ $t = 5 \Rightarrow T = 200\mathrm{e}^{-0.3054\times 5} + 20$ $= 63.43\,(°\mathrm{C})$	Correct DE and correct general solution (3rd line).	Correct DE, correct general solution and correct answer. Units not required.	
(d)	$\int_4^9 18x^{\frac{-3}{2}}\mathrm{d}x = \left[-36x^{\frac{-1}{2}}\right]_4^9$ $= \left[\frac{-36}{\sqrt{x}}\right]_4^9$ $= \frac{-36}{3} - \frac{-36}{2} = 6$	Correct integration.	Correct integration and correct solution.	
(e)	$h = k\int\left(12 + 3\cos\frac{2\pi t}{365}\right)\mathrm{d}t$ $= k\left(12t + \frac{3\times 365}{2\pi}\sin\frac{2\pi t}{365}\right) + c$ $t=0,\ h=84 \ \Rightarrow\ 84 = (12\times 0 + 0) + c$ $\Rightarrow\ c = 84$ At $t = 75$ $\quad 91 = k\left(12\times 75 + \frac{1095}{2\pi}\sin\frac{150\pi}{365}\right) + 84$ $7 = 1067.2k$ $k = 0.00656$ $\therefore h = 0.00656\left(12t + \frac{3\times 365}{2\pi}\sin\frac{2\pi t}{365}\right) + 84$ At $t = 365,\quad h = 0.00656\left(12\times 365 + \frac{3\times 365}{2\pi}\sin\frac{2\pi\times 365}{365}\right) + 84$ $h = 112.7$ cm	Correct integration.	Correct integration and correct values of the constants.	Correct integration, correct values of the constants and correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence	ONE answer demonstrating limited knowledge of integration techniques	ONE correct integration	2u	3u	1r	2r	1t with minor errors ignored	1t

Cut Scores

	Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
Score range	0 – 6	7 – 12	13 – 19	20 – 24