Assessment Schedule – 2016

Calculus: Apply integration methods in solving problems (91579)

Evidence Statement

Q1	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$x^2 - \ln\lvert x\rvert + c$	Correct solution. Absolute value signs not required. c not required.		
(b)	$\frac{1}{3}\sec(3x) + c$	Correct solution. Accept 0.3, 0.33 c not required.		
(c)	$\int 3y\,\mathrm{d}y = \int \cos x\,\mathrm{d}x$ $\frac{3y^2}{2} = \sin x + c$ $x = \frac{\pi}{6},\ y = 1 \Rightarrow \frac{3}{2} = \sin\left(\frac{\pi}{6}\right) + c$ $\frac{3}{2} = \frac{1}{2} + c$ $c = 1$ $\frac{3y^2}{2} = \sin x + 1$ $x = \frac{7\pi}{6} \Rightarrow \frac{3y^2}{2} = \sin\left(\frac{7\pi}{6}\right) + 1$ $= -0.5 + 1$ $= 0.5$ $3y^2 = 1$ $y^2 = \frac{1}{3}$ $y = \pm\sqrt{\frac{1}{3}}$ or $\frac{\pm 1}{\sqrt{3}}$ or ± 0.577	Correct integral. c not required.	Correct solution with correct integral. Accept positive solution only.	
(d)	Area $= \int_0^{1.2} \left(\mathrm{e}^{2x} - \mathrm{e}^{-3x}\right)\mathrm{d}x$ $= \left[\frac{\mathrm{e}^{2x}}{2} + \frac{\mathrm{e}^{-3x}}{3}\right]_0^{1.2}$ $= \left(\frac{\mathrm{e}^{2.4}}{2} + \frac{\mathrm{e}^{-3.6}}{3}\right) - \left(\frac{1}{2} + \frac{1}{3}\right)$ $= 4.69$	Correct integral [inside square brackets]. c not required.	Correct solution with correct integral. Accept any correct numerical substitution (2nd last line)	

(e)	$\frac{\mathrm{d}v}{\mathrm{d}t} = -kvt$ $\int \frac{1}{v}\mathrm{d}v = -\int kt\,\mathrm{d}y$ $\ln v = \frac{-kt^2}{2} + c$ $t = 0 \Rightarrow c = \ln 3000$ $t = 20$ $\ln 2400 = \frac{-k \times 20^2}{2} + \ln 3000$ $\ln 3000 - \ln 2400 = 200k$ $k = \frac{\ln 1.25}{200} = 0.00112$ $t = 96$ $\ln v = \frac{-0.00112 \times 96^2}{2} + \ln 3000$ $= 2.8454$ $v = \mathrm{e}^{2.8454} = 17.2 \text{ mL}$	Correct integral.	Correct integral and correct value of *k*.	Correct integral and correct solution. Accept any correct numerical substitution

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of integration techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q2	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$5x^5 - \frac{10x^3}{3} + x + c$	Correct solution.		
(b)	7	Correct solution.		
(c)	$a(t) = 0.2t + 0.3t^{\frac{1}{2}}$ $v(t) = 0.1t^2 + 0.2t^{\frac{3}{2}} + c$ $t = 4$ $v(4) = 0.1 \times 16 + 0.2 \times 8 + c = 5$ $3.2 + c = 5$ $c = 1.8$ $v(t) = 0.1t^2 + 0.2t^{\frac{3}{2}} + 1.8$ $s(t) = \frac{0.1t^3}{3} + 0.08t^{\frac{5}{2}} + 1.8t + k$ Distance travelled in first 9 seconds $s(9) = \frac{0.1 \times 9^3}{3} + 0.08 \times 9^{\frac{5}{2}} + 1.8 \times 9$ $= 59.94$ m	Correct integral.	Correct solution with correct integral. Accept 59.9, 60. 2nd last line acceptable.	
(d)	$\int_m^{2m} (2x-m)^2 = \left[\frac{1}{6}(2x-m)^3 + c\right]_m^{2m}$ $= \frac{1}{6}\left[(4m-m)^3 - (2m-m)^3\right]$ $= \frac{1}{6}\left[(3m)^3 - m^3\right]$ $= \frac{26m^3}{6}$ $= \frac{13m^3}{3}$ $\therefore \frac{13m^3}{3} = 117$ $m^3 = \frac{117 \times 3}{13} = 27$ $m = 3$	Correct integration [inside square brackets].	Correct solution with correct integration.	

(e)	$(k-1)x^2 = 9 - x^2$ $kx^2 = 9$ $x^2 = \frac{9}{k}$ $x = \frac{\pm 3}{\sqrt{k}}$ $\text{Area} = 2\times\int_0^{\frac{3}{\sqrt{k}}}\left((9-x^2)-(k-1)x^2\right)dx$ $= 2\times\int_0^{\frac{3}{\sqrt{k}}}\left(9-kx^2\right)dx$ $= 2\times\left[9x - \frac{kx^3}{3}\right]_0^{\frac{3}{\sqrt{k}}}$ $= 2\times\left(\frac{27}{\sqrt{k}} - \frac{9}{\sqrt{k}}\right)$ $= \frac{36}{\sqrt{k}}$ $\therefore \frac{36}{\sqrt{k}} = 24$ $\sqrt{k} = 1.5$ $k = 2.25$	$\frac{kx^3}{3} - 9x$	Correct integral. Accept $\left\lvert\frac{kx^3}{3} - 9x\right\rvert$	Correct solution. For E7: $\int_0^{\frac{3}{\sqrt{k}}} = 24$ leads to $\frac{18}{\sqrt{k}} = 24$ $\sqrt{k} = \frac{3}{4}$ $k = \frac{9}{16}$ (E7)

NØ	**N1**	**N2**	**A3**	**A4**	**M5**	**M6**	**E7**	**E8**
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of integration techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q3	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$\int_1^4 \left(4+\frac{k}{x^2}\right)\mathrm{d}x = \left[4x-\frac{k}{x}\right]_1^4$ $=\left(16-\frac{k}{4}\right)-(4-k)$ $=12+\frac{3k}{4}$ $12+\frac{3k}{4}=0$ $k=-16$	Correct solution with correct integration.		
(b)	7.8	Correct solution.		
(c)	$2-x^2=-x$ $x^2-x-2=0$ $(x-2)(x+1)=0$ $x=2,\ -1$ Area $=\int_{-1}^{2}[(2-x^2)-(-x)]\mathrm{d}x$ $=\int_{-1}^{2}(2-x^2+x)\mathrm{d}x$ $=\left[2x-\frac{x^3}{3}+\frac{x^2}{2}\right]_{-1}^{2}$ $=\left(4-\frac{8}{3}+2\right)-\left(-2+\frac{1}{3}+\frac{1}{2}\right)$ $=4.5$	Correct integration [inside square brackets]. Both integrals completed separately is ok for *u*.	Correct solution with correct integration	
(d)	$\int\left(\frac{e^{3x}-x^2}{e^{3x}-x^3}\right)\mathrm{d}x=\frac{1}{3}\int\left(\frac{3e^{3x}-3x^2}{e^{3x}-x^3}\right)\mathrm{d}x$ $=\frac{1}{3}\ln\left\|\left(e^{3x}-x^3\right)\right\|+c$		Correct solution. Absolute value signs not required. Brackets not required.	

(e)	$\sec x \cdot \frac{dy}{dx} = e^{y+\sin x}$ $\frac{1}{\cos x} \cdot \frac{dy}{dx} = e^{y} \cdot e^{\sin x}$ $\int \frac{1}{e^{y}} dy = \int \cos x \cdot e^{\sin x} dx$ $\int e^{-y} dy = \int \cos x \cdot e^{\sin x} dx$ $-e^{-y} = e^{\sin x} + c$ $x = 0, \ y = -1$ $-e^{1} = e^{0} + c$ $c = -e^{1} - 1$ $\therefore -e^{-y} = e^{\sin x} - e - 1$ $e^{-y} = e + 1 - e^{\sin x}$ $x = \frac{\pi}{2}$ $e^{-y} = e + 1 - e$ $e^{-y} = 1$ $y = 0$	Correct integration of 1 side.	Correct integration of both sides.	Correct solution with correct integration. Answer may not be exactly zero if the value of c is evaluated numerically. E.g. $c = -3.7183$ leads to $y = -0.0611$. Accept continuity.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of integration techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0–6	7–13	14–19	20–24