Assessment Schedule – 2018

Calculus: Apply integration methods in solving problems (91579)

Evidence Statement

Q1	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$3x^2 + 4x^{-2} + c$	Correct solution.		
(b)	$\frac{dy}{dx} = e^{2x} + \frac{1}{x}$ $y = \frac{1}{2}e^{2x} + \ln\|x\| + c$ $2 = \frac{1}{2}e^2 + \ln 1 + c$ $c = 2 - \frac{1}{2}e^2 = -1.69$ $y = \frac{1}{2}e^{2x} - \ln\|x\| - 1.69$ $\left(\text{or } y = \frac{1}{2}e^{2x} - \ln\|x\| + 2 - \frac{1}{2}e^2\right)$	Correct solution with correct integration. Absolute value signs not needed.		
(c)	$\int_6^8 \frac{2x-7}{x-5}\,dx = \int_6^8 \left(2 + \frac{3}{x-5}\right)dx$ $= \left[2x + 3\ln\|x-5\|\right]_6^8$ $= (16 + 3\ln(3)) - (12 + 3\ln(1))$ $= 4 + 3\ln 3$ $(= 7.296)$	Correct integration.	Correct solution with correct integration.	
(d)	$\frac{dy}{dx} = \frac{\cos 2x}{e^y}$ $\int e^y\,dy = \int \cos(2x)\,dx$ $e^y = \frac{1}{2}\sin(2x) + c$ $y = 0$ when $x = \frac{\pi}{4}$ $e^0 = \frac{1}{2}\sin\left(\frac{\pi}{2}\right) + c$ $1 = \frac{1}{2} + c$ $c = \frac{1}{2}$ $e^y = \frac{1}{2}\sin(2x) + \frac{1}{2}$	Correct integration.	Correct solution. with correct integration.	

(e)	$y = \frac{1}{2}e^x - \frac{1}{2}$ Area $= \int_0^k \left(\frac{1}{2}e^x - \frac{1}{2}\right)dx$ $= \left[\frac{e^x}{2} - \frac{x}{2}\right]_0^k$ $= \left(\frac{e^k}{2} - \frac{k}{2}\right) - \left(\frac{e^0}{2} - 0\right)$ $= \frac{e^k}{2} - \frac{k}{2} - \frac{1}{2}$ At point Q (k, k) $y = k = \frac{e^k}{2} - \frac{k}{2}$ Area $= \frac{e^k}{2} - \frac{1}{2} - \frac{k}{2}$ $= k - \frac{k}{2}$ $= \frac{k}{2}$	Correct integration.	Correct integration and correct expression for area in terms of k.	Correct solution including correct integration.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of integration techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q2	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$\tan x + \frac{1}{2}\sec 2x + c$	Correct integration.		
(b)	$\int_1^k \sqrt{x}\,\mathrm{d}x = \frac{52}{3}$ $\left[\frac{2}{3}x^{\frac{3}{2}}\right]_1^k = \frac{52}{3}$ $\frac{2}{3}k^{\frac{3}{2}} - \frac{2}{3} = \frac{52}{3}$ $\frac{2}{3}k^{\frac{3}{2}} = \frac{54}{3}$ $k^{\frac{3}{2}} = 27$ $k = 27^{\frac{2}{3}}$ $k = 9$	Correct solution with correct integration.		
(c)	$\int_0^k \left(\cos^2 x - \sin^2 x\right)\mathrm{d}x = \frac{1}{2}$ $\int_0^k \cos(2x)\,\mathrm{d}x = \frac{1}{2}$ $\left[\frac{1}{2}\sin(2x)\right]_0^k = \frac{1}{2}$ $\left[\sin(2x)\right]_0^k = 1$ $\sin 2k - \sin 0 = 1$ $2k = \sin^{-1}(1)$ $k = \frac{\pi}{4}$	Correct integration.	Correct solution with correct integration.	

(d)	$a(t)=\frac{2}{\sqrt{t+1}}=2\left(t+1\right)^{\frac{-1}{2}}$ $v(t)=4\left(t+1\right)^{\frac{1}{2}}+c$ $At\ t=3,\ 9=4\sqrt{4}+c$ $c=1$ $\therefore v(t)=4(t+1)^{\frac{1}{2}}+1$ $\text{Distance }=\int_0^8 (4(t+1)^{\frac{1}{2}}+1)\mathrm{d}t$ $=\left[\frac{8}{3}(t+1)^{\frac{3}{2}}+t\right]_0^8$ $=\left[\frac{8}{3}(9)^{\frac{3}{2}}+8\right]-\left[\frac{8}{3}(1)^{\frac{3}{2}}+0\right]$ $=80-\frac{8}{3}$ $=77\frac{1}{3}\text{ m}$	Correct integration to give *v(t)*.	Correct solution with correct integrations.	
(e)	$\frac{\mathrm{d}m}{\mathrm{d}t}=-k\left(m-10\right)$ $\int\frac{dm}{m-10}=\int -k\mathrm{d}t$ $\ln\lvert m-10\rvert=-kt+c$ $m-10=\mathrm{e}^{-kt+c}$ $m-10=A\mathrm{e}^{-kt}$ $m=A\mathrm{e}^{-kt}+10$ $t=0,\ \ m=140$ $140=10+A\mathrm{e}^{0}$ $A=130$ $m=10+130\mathrm{e}^{-kt}$ $t=3,\ \ m=70$ $70=10+130\mathrm{e}^{-3k}$ $\mathrm{e}^{-3k}=\frac{6}{13}$ $k=\frac{\ln\frac{6}{13}}{-3}=0.2577$ $m=10+130\mathrm{e}^{-0.2577t}$ $50=10+130\mathrm{e}^{-0.2577t}$ $\mathrm{e}^{-0.2577t}=\frac{4}{13}$ $t=\frac{\ln\frac{4}{13}}{-0.2577}=4.57\ (\text{hours})$	Correct integration.	Correct integration and particular solution to the D.E.	Correct solution with correct integration.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of integration techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q3	**Expected coverage**	**Achievement (u)**	**Merit (r)**	**Excellence (t)**
(a)	$\frac{16}{3}x^3 + 2x^2 + 4\ln\lvert x\rvert$	Correct integration. Absolute value signs not needed.		
(b)	3.2	Correct solution.		
(c)	$\int_0^k 3e^{0.5x}dx = \left[6e^{0.5x}\right]_0^k$ $\therefore 6e^{0.5k} - 6 = 75$ $6e^{0.5k} = 81$ $e^{0.5k} = 13.5$ $0.5k = \ln(13.5)$ $k = \frac{\ln(13.5)}{0.5} \quad (= 5.21)$	Correct integration.	Correct solution with correct integration.	
(d)	$x^2 = x^{\frac{1}{3}}$ $x^6 = x$ $x^6 - x = 0$ $x(x^5 - 1) = 0$ $x = 0$ or $x = 1$ Area $= \int_0^1 \left(x^{\frac{1}{3}} - x^2\right)dx$ $= \left[\frac{3}{4}x^{\frac{4}{3}} - \frac{x^3}{3}\right]_0^1$ $= \left(\frac{3}{4} - \frac{1}{3}\right)$ $= \frac{5}{12}$ (or 0.417)	Correct integration.	Correct solution with correct integration.	

(e)	$y=(2x-1)^4$ When $y=0, (2x-1)^4=0$ $x=\frac{1}{2}$ $\therefore$ P is the point $\left(\frac{1}{2},0\right)$ $y=(2x-1)^4$ $\frac{dy}{dx}=8(2x-1)^3$ $\frac{dy}{dx}=8$ When $x=1$ $y-1=8(x-1)$ $y=8x-7$ $=\int_{\frac{1}{2}}^{1}(2x-1)^4\,dx$ Area under the curve between P and Q $=\left[\frac{1}{10}(2x-1)^5\right]_{\frac{1}{2}}^{1}$ $=\frac{1}{10}$ Tangent intercepts x-axis when $8x-7=0$ $x=\frac{7}{8}$ Area under tangent $=\frac{1}{2}\times\frac{1}{8}\times 1=\frac{1}{16}$ Shaded area $=\frac{1}{10}-\frac{1}{16}$ $=\frac{3}{80}$	Correct integral.	Correct area under curve with correct integral.	Correct solution with correct integral.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of integration techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 8	9 – 14	15 – 20	21 – 24