Assessment Schedule – 2020

Calculus: Apply integration methods in solving problems (91579)

Evidence Statement

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$\frac{x^2}{2}+2x+3\ln\lvert x\rvert+c$	Correct integration.		
(b)	$v(t)=0.6\sqrt{t}$ $s(t)=0.4t^{\frac{3}{2}}+c$ $s=5$ when $t=0$, $c=5$ $s(t)=0.4t^{\frac{3}{2}}+5$ $s(16)=0.4\times16^{\frac{3}{2}}+5$ $=30.6$ cm	Correct solution with correct integration.		
(c)	$\int_4^8 \frac{5x-11}{x-3}\,\mathrm{d}x$ $=\int_4^8 5+\frac{4}{x-3}\,\mathrm{d}x$ $=\left[5x+4\ln(x-3)\right]_4^8$ $=\left((5(8)+4\ln 5)-(5(4)+4\ln(1)\right)$ $=26.44$	Correct integration.	Correct solution with correct integration.	

(d)	$x+\frac{3}{x}=4$ $x^2+3=4x$ $x^2-4x+3=0$ $(x-1)(x-3)=0$ $x=1$ or $x=3$ Area $=\int_1^3\left(4-(x+\frac{3}{x})\right)\mathrm{d}x$ $=\int_1^3\left(4-x-\frac{3}{x}\right)\mathrm{d}x$ $=\left[4x-\frac{x^2}{2}-3\ln x\right]_1^3$ $=\left(4(3)-\frac{(3)^2}{2}-3\ln 3\right)-\left(4(1)-\frac{(1)^2}{2}-3\ln 1\right)$ $=0.704$ OR Area $=8-\int_1^3\left(x+\frac{3}{x}\right)\mathrm{d}x$ $=8-\left[\frac{x^2}{2}+3\ln x\right]_1^3$ $=8-\left[\left(\frac{(3)^2}{2}+3\ln 3\right)-\left(\frac{(1)^2}{2}+3\ln 1\right)\right]$ $=0.704$	Correct integration of expression to find area.	Correct solution with correct integration.	

(e)	$\tan x \frac{dy}{dx} = \frac{\sec^2 x}{y}$ $y \frac{dy}{dx} = \frac{\sec^2 x}{\tan x}$ $\int y \, dy = \int \frac{\sec^2 x}{\tan x} dx$ $\frac{y^2}{2} = \ln\lvert\tan x\rvert + c$ $y = 2$ when $x = \frac{\pi}{4}$ $\Rightarrow 2 = \ln\left\lvert\tan\left(\frac{\pi}{4}\right)\right\rvert + c$ $2 = c$ $\therefore \frac{y^2}{2} = \ln\lvert\tan x\rvert + 2$ When $x = \frac{\pi}{3}$, $\frac{y^2}{2} = \ln\left\lvert\tan\left(\frac{\pi}{3}\right)\right\rvert + 2$ $\frac{y^2}{2} = 0.5493 + 2$ $y^2 = 5.099$ $y = \pm 2.26$	Correct integration on either side with separation of variables.	Correct integration.	Correct solution with correct integration. $y = 2.26$ is E7.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of integration techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$\pi x + \frac{2}{x} + c$	Correct integration.		
(b)	6	Correct solution.		
(c)	$\int_1^k 9\sqrt{3x-2}\,\mathrm{d}x = \int_1^k 9(3x-2)^{\frac{1}{2}}\,\mathrm{d}x$ $= \left[9 \times \frac{2}{9}(3x-2)^{\frac{3}{2}}\right]_1^k$ $= \left[2(3x-2)^{\frac{3}{2}}\right]_1^k$ $= 2(3k-2)^{\frac{3}{2}} - 2 = 126$ $2(3k-2)^{\frac{3}{2}} = 128$ $(3k-2)^{\frac{3}{2}} = 64$ $3k-2 = 16$ $k = 6$	Correct integration of $9\sqrt{3x-2}$.	Correct solution with correct integration.	
(d)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{y} \cdot \cos 4x$ $\int \frac{1}{\sqrt{y}} dy = \int \cos 4x \mathrm{d}x$ $2\sqrt{y} = \frac{\sin 4x}{4} + c$ $y = 1$ when $x = \frac{\pi}{8}$ $2 = \frac{\sin \frac{\pi}{2}}{4} + c$ $2 = \frac{1}{4} + c$ $c = 1.75$ $2\sqrt{y} = \frac{\sin 4x}{4} + 1.75$ $x = \frac{\pi}{4}$ $2\sqrt{y} = \frac{\sin \pi}{4} + 1.75$ $\sqrt{y} = \frac{7}{8}$ $y = \frac{49}{64} (= 0.765625)$	Correct integration of either side.	Correct solution with correct integrations.	

(e)	$x+2\sqrt{x}-3=0$ $(\sqrt{x}+3)(\sqrt{x}-1)=0$ $x=1$ $\text{Area}=\left\lvert\int_0^1 (x+2\sqrt{x}-3)\,\mathrm{d}x\right\rvert+\int_1^4 (x+2\sqrt{x}-3)\,\mathrm{d}x$ $=\left\lvert\left[\frac{x^2}{2}+\frac{4}{3}x^{\frac{3}{2}}-3x\right]_0^1\right\rvert+\left[\frac{x^2}{2}+\frac{4}{3}x^{\frac{3}{2}}-3x\right]_1^4$ $=\left\lvert\left(\frac{1}{2}+\frac{4}{3}-3\right)-0\right\rvert+\left[\left(8+\frac{32}{3}-12\right)-\left(\frac{1}{2}+\frac{4}{3}-3\right)\right]$ $=\frac{7}{6}+\left[\frac{20}{3}+\frac{7}{6}\right]$ $=\frac{27}{3}$ $=9$	Correct integration of expression.	Correct integration plus signed area clearly shown and used correctly.	Correct solution with correct integral and correct use of signed area.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of integration techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$\frac{1}{2}\sec 2x + c$	Correct integration.		
(b)	$y = \frac{1}{2}\sin 2x + c$ $1 = \frac{1}{2} \times \frac{1}{2} + c$ $c = \frac{3}{4}$ $\Rightarrow y = \frac{1}{2}\sin 2x + \frac{3}{4}$ $y = \frac{1}{2}\sin\frac{\pi}{2} + \frac{3}{4}$ $y = \frac{5}{4} (= 1.25)$	Correct solution with correct integration.		
(c)	$\frac{dv}{dt} = a(t) = t + e^{0.2t}$ $v = \frac{1}{2}t^2 + 5e^{0.2t} + c$ $t = 0, v = 8$ $8 = 5 + c$ $c = 3$ $\therefore v = \frac{1}{2}t^2 + 5e^{0.2t} + 3$ $t = 10 \quad v = \frac{1}{2}10^2 + 5e^2 + 3$ $= 89.95 \text{ m s}^{-1}$	Correct integration.	Correct solution with correct integration.	

(d)	$\frac{dN}{dt} = kN$ $\int \frac{1}{N}\,dN = \int k\,dt$ $\ln N = kt + c\;[\ln(CN) = kt]$ $N = Ae^{kt}$ $0.5 = e^{5.6k}$ $k = \frac{\ln 0.5}{5.6} = -0.1238$ $N = Ae^{-0.1238t}$ $0.05 = e^{-0.1238t}$ $\ln(0.05) = -0.1238t$ $t = \frac{\ln(0.05)}{-0.1238}$ $= 24.2$ days OR $\ln N = kt + c$ $t = 0, N = 1 \Rightarrow c = 0$ $\ln N = kt$ $\ln(0.5) = 5.6k$ $k = \frac{\ln(0.5)}{5.6} = -0.1238$ $\ln N = -0.1238t$ $\ln(0.05) = -0.1238t$ $t = \frac{\ln(0.05)}{-0.1238}$ $= 24.2$ days OR $\ln N = kt + c$ $t = 0, N = 100 \Rightarrow c = \ln(100)$ $\ln N = kt + \ln(100)$ $\ln(50) = 5.6k + \ln(100)$ $k = \frac{\ln(50) - \ln(100)}{5.6} = -0.1238$ $\ln N = -0.1238t + \ln(100)$ $\ln(5) = -0.1238t + \ln(100)$ $t = \frac{\ln(5) - \ln(100)}{-0.1238}$ $= 24.2$ days	General solution of DE clearly shown with 2 constants.	Correct solution with general solution of DE clearly shown.	

(e)	Area $= \int_0^{\frac{\pi}{2}} \left(\cos x - \cos^3 x\right) dx$ $= \int_0^{\frac{\pi}{2}} \cos x\left(1 - \cos^2 x\right) dx$ $= \int_0^{\frac{\pi}{2}} \cos x\left(\sin^2 x\right) dx$ $= \left[\frac{1}{3}\sin^3 x\right]_0^{\frac{\pi}{2}}$ $= \left(\frac{1}{3}\left(\sin(\frac{\pi}{2})\right)^3 - \frac{1}{3}\left(\sin(0)\right)^3\right)$ $= \frac{1}{3}$		Correct integral.	Correct solution with correct integral.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of integration techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 8	9 – 14	15 – 19	20 – 24