Assessment Schedule – 2022

Calculus: Apply integration methods in solving problems (91579)

Evidence Statement

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$4\ln\lvert x\rvert - \tan x + c$	Correct integral.		
(b)	6.4	Correct solution.		
(c)	$\int_0^{\frac{\pi}{4}} \sin^2(2x)\,\mathrm{d}x$ $= \int_0^{\frac{\pi}{4}} \left(\frac{1}{2} - \frac{1}{2}\cos(4x)\right)\mathrm{d}x$ $= \left[\frac{x}{2} - \frac{1}{8}\sin 4x\right]_0^{\frac{\pi}{4}}$ $= \left(\frac{\pi}{8} - \frac{1}{8}\sin\pi\right) - (0-0)$ $= \frac{\pi}{8}$	Correct integral.	Correct solution with correct integral. Accept 0.393.	
(d)	$y = \frac{4}{\sqrt{3x-2}} = 4(3x-2)^{\frac{-1}{2}}$ Area $= \int_1^k 4(3x-2)^{\frac{-1}{2}}\,\mathrm{d}x$ $= \left[\frac{8}{3}(3x-2)^{\frac{1}{2}}\right]_1^k$ $= \frac{8}{3}\sqrt{3k-2} - \frac{8}{3}\times 1 = 8$ $\sqrt{3k-2} - 1 = 3$ $\sqrt{3k-2} = 4$ $3k - 2 = 16$ $k = 6$	Correct integral.	Correct solution with correct integral.	

(e)	Limits of integration $(e^x)^2 = 3e^x + 10$ $(e^x)^2 - 3e^x - 10 = 0$ $(e^x - 5)(e^x + 2) = 0$ $e^x = 5$ or $e^x = -2$ no $x = \ln 5$ $\text{Area} = \int_0^{\ln 5} (3e^x + 10)\,dx - \int_0^{\ln 5} e^{2x}\,dx$ $= \int_0^{\ln 5} (3e^x + 10 - e^{2x})\,dx$ $= \left[3e^x + 10x - \frac{e^{2x}}{2}\right]_0^{\ln 5}$ $= \left(3e^{\ln 5} + 10\ln 5 - \frac{e^{2\ln 5}}{2}\right) - \left(3 + 0 - \frac{1}{2}\right)$ $= (15 + 10\ln 5 - 12.5) - 2.5$ $= 10\ln 5$	Correct integral expression for area with correct limits and e^{2x}.	Correct integration.	Correct solution with correct integration.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$\frac{e^{3x}}{3}-\frac{2}{3}x^{\frac{3}{2}}+c$	Correct integral.		
(b)	$\int_1^k \frac{2}{\sqrt{x}}\,dx=8$ $\int_1^k 2x^{\frac{-1}{2}}\,dx=8$ $\left[4\sqrt{x}\right]_1^k=8$ $4\sqrt{k}-4=8$ $4\sqrt{k}=12$ $\sqrt{k}=3$ $k=9$	Correct solution with correct integral.		
(c)	$\frac{dy}{dx}=\frac{1}{3y^2(x-1)}$ $\int 3y^2\,dy=\int\frac{1}{x-1}\,dx$ $y^3=\ln\|x-1\|+c$ $x=2,\ y=-1$ $-1=\ln(1)+c$ $c=-1$ When $y=1$ $1=\ln(x-1)-1$ $\ln(x-1)=2$ $x=e^2+1\,(=8.389)$	Correct integration.	Correct solution with correct integral.	
(d)	$a(t)=0.9e^{0.3t}$ $v(t)=3e^{0.3t}+c$ When $t=2, v=10$ $10=3e^{0.6}+c$ $c=4.534$ $v(t)=3e^{0.3t}+4.534$ Distance travelled in 5th second of motion $=\int_4^5(3e^{0.3t}+4.534)\,dt$ $=\left[10e^{0.3t}+4.534t\right]_4^5$ $=(10e^{1.5}+4.534\times 5)-(10e^{1.2}+4.534\times s4)$ $=16.1497$ m	Correct equation for v.	Correct solution with correct integrals.	

(e)	$\frac{dh}{dt}=\frac{-1}{4}\sqrt{(h-6)^3}$ $\int (h-6)^{\frac{-3}{2}}\,dh=\int \frac{-1}{4}\,dt$ $-2(h-6)^{\frac{-1}{2}}=\frac{-t}{4}+c$ $(h-6)^{\frac{-1}{2}}=\frac{t}{8}+k$ $\frac{1}{\sqrt{h-6}}=\frac{t}{8}+k$ When $t=0, h=150$ $\frac{1}{\sqrt{144}}=k$ $k=\frac{1}{12}$ $\frac{1}{\sqrt{h-6}}=\frac{t}{8}+\frac{1}{12}$ $h=15$ $\frac{1}{\sqrt{9}}=\frac{t}{8}+\frac{1}{12}$ $\frac{t}{8}=\frac{1}{4}$ $t=2$	Correct integral wrt *h*.	Correct solution of DE.	Correct solution of problem with all integrals correct.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	**Achievement (u)**	**Merit (r)**	**Excellence (t)**
THREE (a)	$\frac{1}{8}(2x+5)^4+c$	Correct integral.		
(b)	8.92	Correct solution.		
(c)	$\int_5^8 \frac{4x-5}{x-3}\mathrm{d}x$ $\frac{4x-5}{x-3}=4+\frac{7}{x-3}$ $\int_5^8 \frac{4x-5}{x-3}\mathrm{d}x=\int_5^8\left(4+\frac{7}{x-3}\right)\mathrm{d}x$ $=\left[4x+7\ln\lvert x-3\rvert\right]_5^8$ $=(32+7\ln 5)-(20+7\ln 2)$ $=18.41$	Correct integral.	Correct solution with correct integral.	
(d)	Limits of integration $x=x+\cos x$ $\cos x=0$ $x=\frac{-\pi}{2},\frac{\pi}{2}$ Area $=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}(x+\cos x)\mathrm{d}x-\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}x\,\mathrm{d}x$ $=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}(\cos x)\mathrm{d}x$ $=\left[\sin x\right]_{\frac{-\pi}{2}}^{\frac{\pi}{2}}$ $=1--1$ $=2$	Correct integral.	Correct solution with correct integral.	
(e)	Area of rectangle $=2k\times\frac{2}{k}=4$ Area under curve $=\int_k^{3k}\frac{2}{x}\mathrm{d}x$ $=\left[2\ln x\right]_k^{3k}$ $=2\ln 3k-2\ln k$ $=\ln 9k^2-\ln k^2$ $=\ln\left(\frac{9k^2}{k^2}\right)$ $=\ln 9$ Shaded area $=4-\ln 9$ $a=4, b=-1$, and $c=9$.	Correct integration.	Correct area under curve with correct integration.	Correct solution. Accept $4-\ln 9$. Accept $4-2\ln 3$.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 14	15 – 20	21 – 24