Assessment Schedule – 2023

Calculus: Apply integration methods in solving problems (91579)

Evidence Statement

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$= \frac{3x^2}{2} + 2x + \frac{1}{3}\ln(3x+2) + c$	• Correct integral $+\,c$ not required.		
(b)	$d = \int \sec^2 t\,\mathrm{d}t$ $\Rightarrow d = \tan t + c$ $t = 0$ and $d = 3$ give $3 = \tan 0 + c$ $\Rightarrow c = 3$ i.e. $d = \tan t + 3$ $t = \frac{\pi}{4} \Rightarrow d = \tan\frac{\pi}{4} + 3$ $d = 1 + 3 = 4$ km	• Correct solution, with evidence of correct integration.		
(c)	For point of intersection between the two curves, solve $\sqrt{x} = \frac{x^2}{8}$, Giving $x = 0$ and $x = 4$. Area $= \int_0^4 \left(\sqrt{x} - \frac{x^2}{8}\right) \mathrm{d}x$ $= \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{x^3}{3\times 8}\right]_0^4$ $= \left[\frac{16}{3} - \frac{64}{24}\right] - [0] = \frac{8}{3}$ units^2	• Correct integral.	• Correct solution, with evidence of correct integration.	
(d)	$\frac{\mathrm{d}y}{\mathrm{d}x} = y(2x - 3x^2)$ Separating the variables gives: $\int \frac{1}{y}\,\mathrm{d}y = \int (2x - 3x^2)\,\mathrm{d}x$ $\ln y = x^2 - x^3 + c$ #(1) $x = 2$ and $y = 1$ gives: $\ln 1 = 4 - 8 + c \Rightarrow c = 4$ i.e. $\ln y = x^2 - x^3 + 4$ $x = 1 \Rightarrow \ln y = 1 - 1 + 4$ $\Rightarrow \ln y = 4$ $y = \mathrm{e}^4 = 54.6$	• Reaching stage #(1) of the solution.	• Correct solution, with evidence of correct integration.	

(e)	Intersection points are (1,3) and (10,0). Area of Triangle A $= \frac{1}{2} \times 1 \times 3 = 1.5$ (**or** by integration) Area of $B = \int_1^{10} \sqrt{10-x}\,dx$ $= \int_1^{10} (10-x)^{\frac{1}{2}}\,dx$ $= \left[-\frac{2}{3}(10-x)^{\frac{3}{2}}\right]_1^{10}$ $= -\frac{2}{3}[0-27] = 18$ Area of $C = \int_0^{10} -\sin\frac{\pi x}{10}\,dx$ $= \left[\frac{10}{\pi}\cos\frac{\pi x}{10}\right]_0^{10}$ $= \frac{10}{\pi}[\cos\pi - \cos 0]$ $= \frac{10}{\pi}[-1-1]$ $= -\frac{20}{\pi}$ i.e. Actual area will be $\frac{20}{\pi}$. Total area $1.5 + 18 + \frac{20}{\pi}$ $= 25.866$ units^2	• Correct integration of one of the expressions.	Correct evaluation of Area B and C.	E 7 Solution with one minor error. E 8 Correct area, with evidence of correct integrations.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$\int 4e^{2x-1}\,dx = 2e^{2x-1} + c$	• Correct integral, +c not required.		
(b)	$y = \int (4x+1)^{-\frac{1}{2}}\,dx$ $y = \frac{1}{4}\frac{(4x+1)^{\frac{1}{2}}}{\frac{1}{2}} + c$ $y = \frac{1}{2}(4x+1)^{\frac{1}{2}} + c$ $x = 6$ and $y = 7.5$ gives: $7.5 = \frac{1}{2} \times 25^{\frac{1}{2}} + c \Rightarrow c = 5$ i.e. $y = \frac{1}{2}(4x+1)^{\frac{1}{2}} + 5$	• Correct solution, with evidence of correct integration.		
(c)	$\int_2^k \left(3 + \frac{6}{2x-3}\right)dx = 3k$ $\Rightarrow \left[3x + 3\ln(2x-3)\right]_2^k = 3k$ $\Rightarrow 3k + 3\ln(2k-3) - 6 = 3k$ $\Rightarrow 3\ln(2k-3) = 6$ $\Rightarrow \ln(2k-3) = 2$ $\Rightarrow 2k - 3 = e^2$ $\Rightarrow k = \frac{e^2+3}{2} = 5.1945$	• Correct integral.	• Correct solution, with evidence of correct integration.	
(d)	$= -\frac{1}{2}\int \frac{-2\cos 2x - 2\sin 2x}{\cos 2x - \sin 2x}dx$ $= -\frac{1}{2} \times \ln(\cos 2x - \sin 2x) + c$	• Integral, but with missing negative sign.	• Correct integral.	

<table>
<tr>
<td>(e)</td>
<td>
$\frac{dv}{dt} = -kv^2$
$\int \frac{1}{v^2}\,dv = \int -k\,dt$
$\int v^{-2}\,dv = \int -k\,dt$
$\frac{v^{-1}}{-1} = -kt + c$ #(1)
$\frac{-1}{v} = -kt + c$
$\frac{1}{v} = kt + d$
$t = 1$ and $v = p$ gives:
$\frac{1}{p} = k + d$ eq(1)
$t = 2$ and $v = \frac{4}{5}p$ gives:
$\frac{1}{\frac{4}{5}p} = 2k + d$
$\frac{5}{4p} = 2k + d$ eq(2)
$\text{eq}(2) - \text{eq}(1) \Rightarrow \frac{5}{4p} - \frac{1}{p} = k$
i.e. $k = \frac{1}{4p}$
Then eq(1) gives $\frac{1}{p} = \frac{1}{4p} + d$
$d = \frac{3}{4p}$
i.e. $\frac{1}{v} = \frac{1}{4p}t + \frac{3}{4p}$
$t = 0$ gives $\frac{1}{v} = 0 + \frac{3}{4p}$ #(2)
i.e. $v = \frac{4}{3}p$
i.e. The container had $\frac{4}{3}p$ litres of chocolate initially.
</td>
<td>• Reaching stage #(1) of the solution.</td>
<td>• Finding the value of k, in terms of p.
OR
Finding the value of d, in terms of p.</td>
<td>E 7
Reaching stage #(2) of the solution.
OR
Solution with one minor error

E 8
Correct volume of chocolate initially, with evidence of correct integrations.</td>
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NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$\text{Area} \approx \frac{1}{3} \times 3 \times [10 + 14 + 4(13 + 15) + 2(16)]$ $= 1 \times [24 + 112 + 32]$ $= 168\ \text{m}^2$ *Units not required.*	• Correct solution.		
(b)	$= \int \left(1 - \frac{3}{x^{\frac{1}{2}}}\right) dx$ $= \int \left(1 - 3x^{-\frac{1}{2}}\right) dx$ $= x - \frac{3x^{\frac{1}{2}}}{\frac{1}{2}} + c$ $= x - 6x^{\frac{1}{2}} + c$	• Correct integral, +c not required.		
(c)	$= \int_0^{\frac{\pi}{3}} 5 \sin 3x \sin x\, dx$ $= \frac{5}{2} \int_0^{\frac{\pi}{3}} 2 \sin 3x \sin x\, dx$ $= \frac{5}{2} \int_0^{\frac{\pi}{3}} \cos 2x - \cos 4x\, dx$ $= \frac{5}{2} \left[\frac{1}{2} \sin 2x - \frac{1}{4} \sin 4x\right]_0^{\frac{\pi}{3}}$ $= \frac{5}{2}[0.6495 - 0]$ $= \frac{5}{2} \times 0.6495$ $= 1.6238\ \text{units}^2$ or $\frac{15\sqrt{3}}{16}$	• Correct integral.	• Correct area, with evidence of correct integration.	
(d)	$v = \int \frac{e^{2t}}{4e^{2t} - 3} dt$ $v = \frac{1}{8} \int \frac{8e^{2t}}{4e^{2t} - 3} dt$ $v = \frac{1}{8} \ln(4e^{2t} - 3) + c$ $t = 0$ and $v = 5$ gives $5 = \frac{1}{8} \ln 1 + c \Rightarrow c = 5$ i.e. $v = \frac{1}{8} \ln(4e^{2t} - 3) + 5$ $t = 4$ gives $v = 6.1732\ \text{m s}^{-1}$	• Correct equation for v, with evidence of correct integration.	• Correct solution, with evidence of correct integration.	

(e)	$\int \frac{1+y}{1-y^2}\,dy = -\int \frac{1-x}{1-x^2}\,dx$ $\int \frac{1+y}{(1-y)(1+y)}\,dy = -\int \frac{1-x}{(1-x)(1+x)}\,dx$ $\int \frac{1}{(1-y)}\,dy = -\int \frac{1}{(1+x)}\,dx$ $-\ln(1-y) = -\ln(1+x) + c$ #(1) $y = 0$ and $x = 2$ gives: $-\ln 1 = -\ln 3 + c$ $\Rightarrow c = \ln 3 = 1.09863$ i.e. $-\ln(1-y) = -\ln(1+x) + \ln 3$ #(2) $\ln(1-y) = \ln(1+x) - \ln 3$ $\ln(1-y) = \ln\left(\frac{1+x}{3}\right)$ $1-y = \frac{1+x}{3}$ #(3) $1 - \frac{1}{3} - \frac{x}{3} = y$ $y = \frac{2-x}{3}$ $x = 6$ gives $y = \frac{-4}{3}$ OR Alternative solution from #(2) onwards $-\ln(1-y) = -\ln(1+x) + 1.09863$ $\ln(1-y) = \ln(1+x) - 1.09863$ $\ln(1-y) - \ln(1+x) = -1.09863$ $\ln\left(\frac{1-y}{1+x}\right) = -1.09863$ $\frac{1-y}{1+x} = e^{-1.09863}$ #(3) $\frac{1-y}{1+x} = \frac{1}{3}$ $y = \frac{2-x}{3}$ $x = 6$ gives $y = \frac{-4}{3}$	• Correct integral for either of the two integrals at stage #(1) of the solution.	• Correct solution of the differential equation, i.e. stage #(2) of the solution.	E 7 Solution with one minor error. E 8 Correct solution, with evidence of correct integrations.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 6	7 – 12	13 – 8	19 – 24