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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

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SUPERVISOR'S USE ONLY

Level 3 Calculus, 2013

91579 Apply integration methods in solving problems

9.30 am Wednesday 13 November 2013
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–11 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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You are advised to spend 60 minutes answering the questions in this booklet.

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QUESTION ONE

For parts (a) and (b) find each integral.

Remember the constant of integration.

(a) $\int \frac{2x-1}{x^3}\,dx$

(b) $\int (\pi - e^{2x})\,dx$

(c) An object is moving in a straight line.

The velocity of the object is given by $v = 6 - \frac{5}{t+1}$, where t is the time measured in seconds from when the object started moving, and v is the velocity measured in metres per second.

How far does the object travel in its 4th second of motion?

Give the result of any integration needed to solve this problem.

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(d) A property owner assumes that the rate of increase of the value of his property at any time is proportional to the value, $\$V$, of the property at that time.

(i) Write the differential equation that expresses this statement.

(ii) The property was valued at \$365 000 in May 2012, and at \$382 000 in November 2013.

Solve the differential equation in (i) to find the price the owner would have paid in May 2007 when he bought his house, given his assumption is accurate.

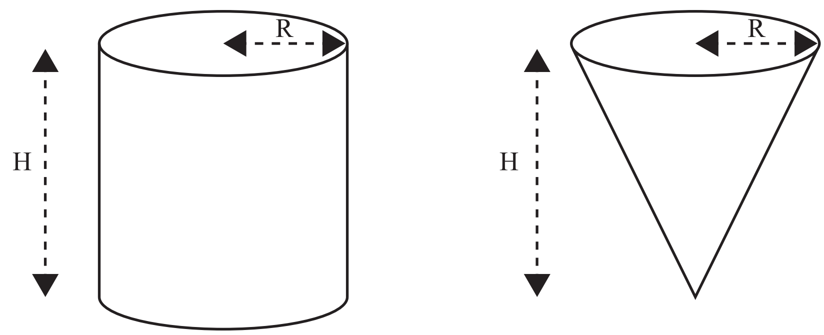
(e) The energy required to pump water out of a tank with a circular cross-section and height H is given by:

$$E = \int_0^{\mathrm{H}} \mathrm{k}(\mathrm{H} - h)A(h)\,\mathrm{d}h$$

where k is a constant
h is the height of the water in the tank at any instant
r is the radius of the water surface at this instant
$A(h)$ is the area of the surface of water at this instant.

A cylindrical tank and a conical tank are both full of water.

Both have height H, and the radius at the top of both tanks is R.



Show that the energy required to empty the conical tank is one sixth the energy required to empty the cylindrical tank.

Give the results of any integration needed to solve this problem.

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QUESTION TWO

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(a) Use the values given in the table below to find the best approximation to $\int_1^2 f(x)\,dx$, using the Trapezium Rule.

x	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$	1.41	1.56	1.72	1.89	2.06	2.24

(b) Find $\int_1^k 3\sqrt{x}\,dx$, giving your answer in terms of k.

(c) Use integration to find the area enclosed between the graphs of the functions $3y = x^2$ and $y = 2x$.

You must use calculus and give the result of any integration needed to solve this problem.

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(d) A large tank initially contains 20 litres of diesel.

The tank is being filled at a rate of $\frac{400}{(t+2)^2}$ litres per minute, where t is the time in minutes since the filling started.

How much diesel will be in the tank 6 minutes after filling started?

Give the result of any integration needed to solve this problem.

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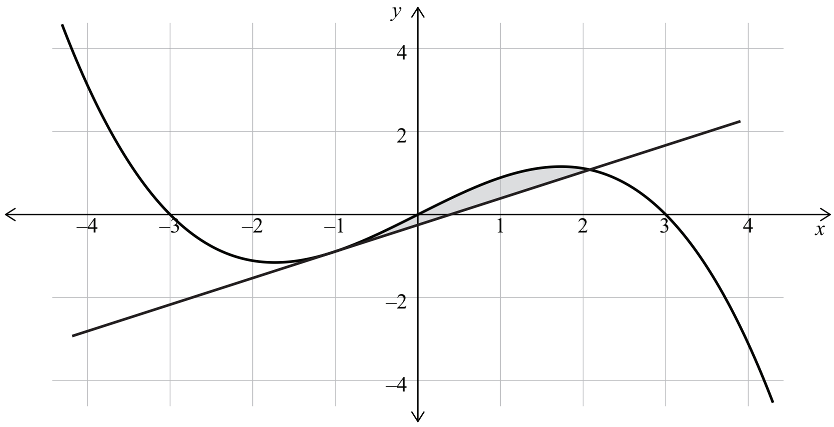
(e) A curve $y = f(x)$, which passes through the origin, is shown on the graph below.

Its gradient at any point is given by the equation $f'(x) = 1 - \frac{1}{3}x^2$.

The line on the graph is the tangent to the curve at $x = -1$.

Find the shaded area.



You must use calculus and give the result of any integration needed to solve this problem.

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QUESTION THREE

(a) Find $\int \operatorname{cosec}(2x)\cot(2x)\,\mathrm{d}x$.

(b) Find the area enclosed between the graph of $y = \sin(2x)$, the x-axis, and the lines $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$.

Give the result of any integration needed to solve this problem.

(c) Find the value of m such that $\int_{\mathrm{m}}^{2\mathrm{m}} \frac{2x+5}{x^2+5x}\,\mathrm{d}x = \ln 3$.

Give the result of any integration needed to solve this problem.

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(d) The motion of an object is described by the equation $\frac{\mathrm{d}v}{\mathrm{d}t} = -\mathrm{k}v^2$, where v is the velocity of the object in metres per second, t is the time in seconds, and k is a constant.

The intial velocity of the object is u metres per second.

Show that, after one second, the velocity of the object is $v = \frac{u}{\mathrm{k}u + 1}$.

Give the result of any integration needed to solve this problem.

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(e) Ben leaves his cup of coffee on the table to cool. The room's temperature remains constant at 18°C.

The rate at which the temperature of the coffee changes at any instant is proportional to the difference between the temperature of the coffee and the room temperature at that instant.

(i) Write the differential equation that expresses this statement.

(ii) The temperature of the coffee when it is poured is 65°C.

After 2 minutes, its temperature is 62°C.

Ben can drink the coffee when its temperature is 54°C.

Solve the differential equation in (i) to find how long from when it was poured, that Ben has to wait before he can first drink the coffee.

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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