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NZQA

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QUALIFY FOR THE FUTURE WORLD
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Level 3 Calculus, 2018

91579 Apply integration methods in solving problems

9.30 a.m. Tuesday 13 November 2018
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

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(a) Find $\int \left(6x - \frac{8}{x^3}\right) \mathrm{d}x$

(b) Solve the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{2x} + \frac{1}{x}$, given that when $x = 1$, $y = 2$.

(c) Find $\int_6^8 \frac{2x-7}{x-5} \mathrm{d}x$

You must use calculus and show the results of any integration needed to solve the problem.

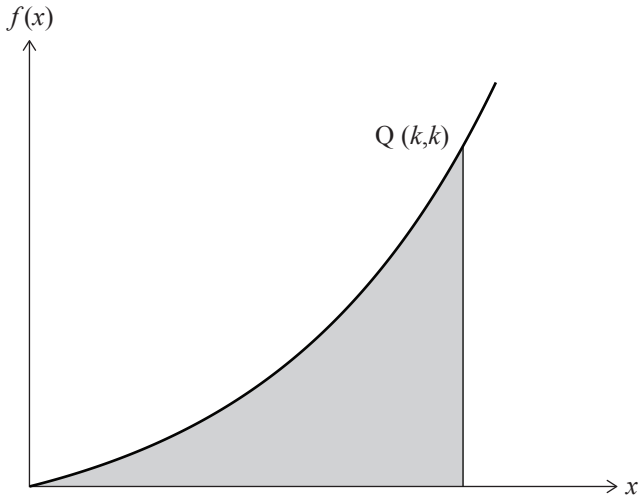
ASSESSOR'S USE ONLY

(d) Solve the differential equation $\frac{dy}{dx} = \frac{\cos 2x}{e^y}$ given that $y = 0$ when $x = \frac{\pi}{4}$.

You must use calculus and show the results of any integration needed to solve the problem.

ASSESSOR'S USE ONLY

(e) The diagram below shows the graph of the function $f(x) = \frac{1}{2}\left(e^x - 1\right)$.



The point Q (k,k) lies on the curve.
The shaded region in the diagram is bounded by the curve, the x -axis and the line $x = k$.

Show that the shaded region has an area of $\frac{1}{2}k$.

You must use calculus and show the results of any integration needed to solve the problem.

QUESTION TWO

ASSESSOR'S USE ONLY

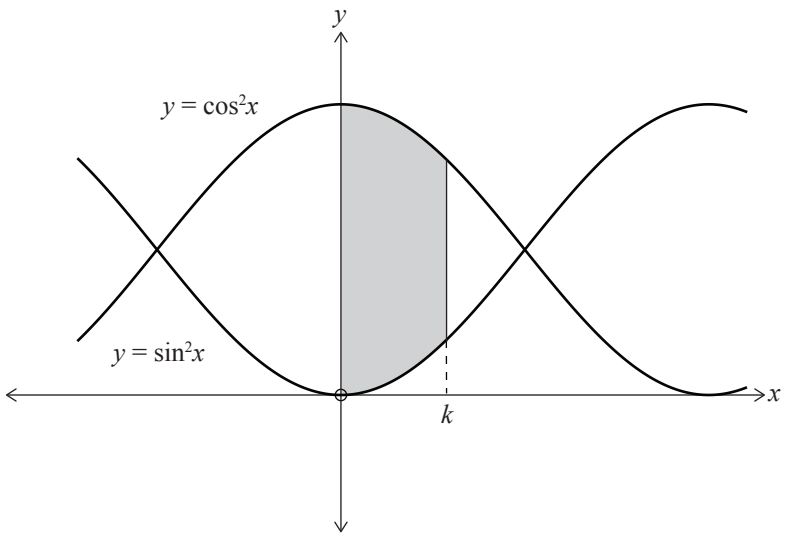
(a) Find $\int \left(\sec^2 x + \sec 2x \tan 2x \right) \mathrm{d}x$

(b) Find the value of k, given that $\int_1^k \sqrt{x}\, \mathrm{d}x = \frac{52}{3}$.

You must use calculus and show the results of any integration needed to solve the problem.

ASSESSOR'S USE ONLY

(c) The diagram below shows the graphs of the functions $y = \cos^2 x$ and $y = \sin^2 x$.



Find the value of k such that $\int_0^k \left(\cos^2 x - \sin^2 x\right) \mathrm{d}x = \frac{1}{2}$.

You must use calculus and show the results of any integration needed to solve the problem.

ASSESSOR'S USE ONLY

(d) An object's acceleration can be modelled by the equation $a(t) = \dfrac{2}{\sqrt{t+1}}$, where $t \geq 0$.

where a is the acceleration of the object in m s^{-2}
and t is the time in seconds from the start of timing.

The object has a velocity of 9 m s^{-1} when $t = 3$.

How far did the object travel in the first 8 seconds of its timed motion?

You must use calculus and show the results of any integration needed to solve the problem.

ASSESSOR'S USE ONLY

(e) The mass, m grams, of a burning candle t hours after it was first lit can be modelled by the differential equation

$$\frac{dm}{dt} = -k(m-10) \text{ where } k > 0 \text{ and } m \geq 10.$$

The initial mass of the candle was 140 grams.

3 hours later the mass of the candle had halved.

Find the length of time it will take for the mass of the candle to reduce to 50 grams.

You must use calculus and show the results of any integration needed to solve the problem.

QUESTION THREE

ASSESSOR'S USE ONLY

(a) Find $\int\left(\left(4x\right)^2+4x+\frac{4}{x}\right)\mathrm{d}x$

(b) Use the values given in the table below to find an approximation to $\int_0^3 f(x)\mathrm{d}x$, using Simpson's Rule.

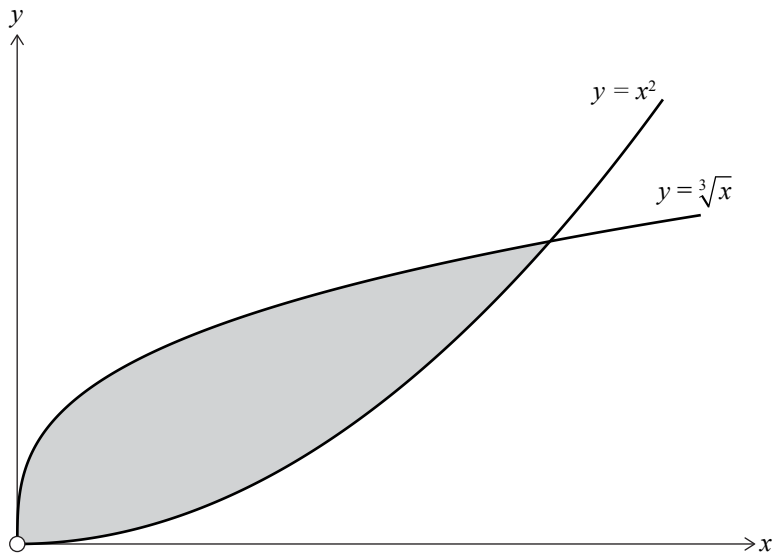
x	0	0.5	1	1.5	2	2.5	3
$f(x)$	0.3	0.75	1.1	1.35	1.6	1.15	0.5

(c) Find the value of k given that $\int_0^k 3\mathrm{e}^{0.5x}\,\mathrm{d}x = 75$.

Question Three continues on the following page.

ASSESSOR'S USE ONLY

(d) The diagram below shows the graphs of the functions $y = x^2$ and $y = \sqrt[3]{x}$.

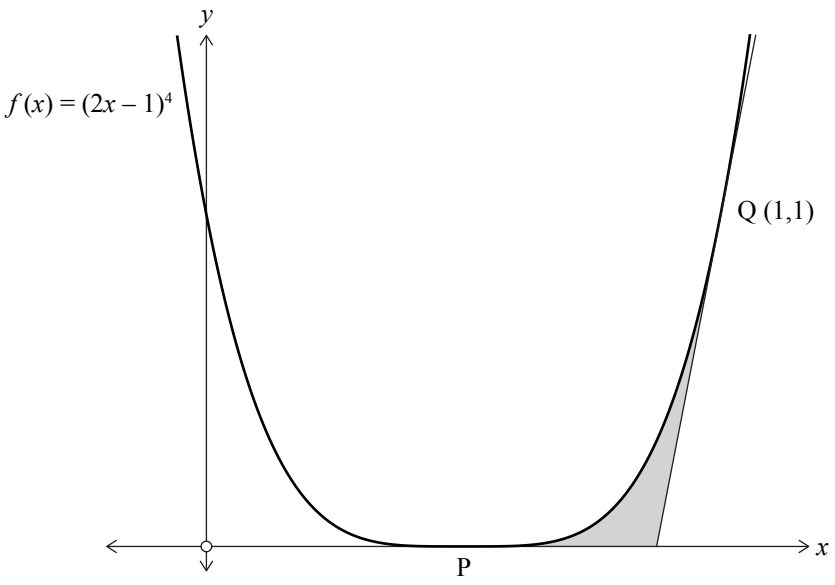


Find the area of the region between the graphs (shown shaded in the diagram).

You must use calculus and show the results of any integration needed to solve the problem.

(e) The diagram below shows the graph of the function $f(x) = (2x - 1)^4$.

ASSESSOR'S USE ONLY



The curve meets the x-axis at P and the line on the graph is a tangent to the curve at the point Q (1,1).

Find the area of the region bounded by the curve, the x-axis, and the tangent to the curve at Q (shown shaded in the diagram).

You must use calculus and show the results of any integration needed to solve the problem.

Extra paper if required.
Write the question number(s) if applicable.

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