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NZQA

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SUPERVISOR'S USE ONLY

Level 3 Calculus, 2019

91579 Apply integration methods in solving problems

9.30 a.m. Tuesday 26 November 2019
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

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(a) Find $\int\left(2+\frac{2}{\sqrt{x}}\right)\mathrm{d}x$.

(b) Use the values given in the table below to find an approximation to $\int_2^5 f\left(x\right)\mathrm{d}x$, using the Trapezium Rule.

x	2	2.5	3	3.5	4	4.5	5
$f(x)$	0.6	1.1	1.7	2.6	3.2	3.4	2.6

(c) Find $\int_0^{\frac{\pi}{12}}\cos 4x.\cos 2x\,\mathrm{d}x$.

You must use calculus and show the results of any integration needed to solve the problem.

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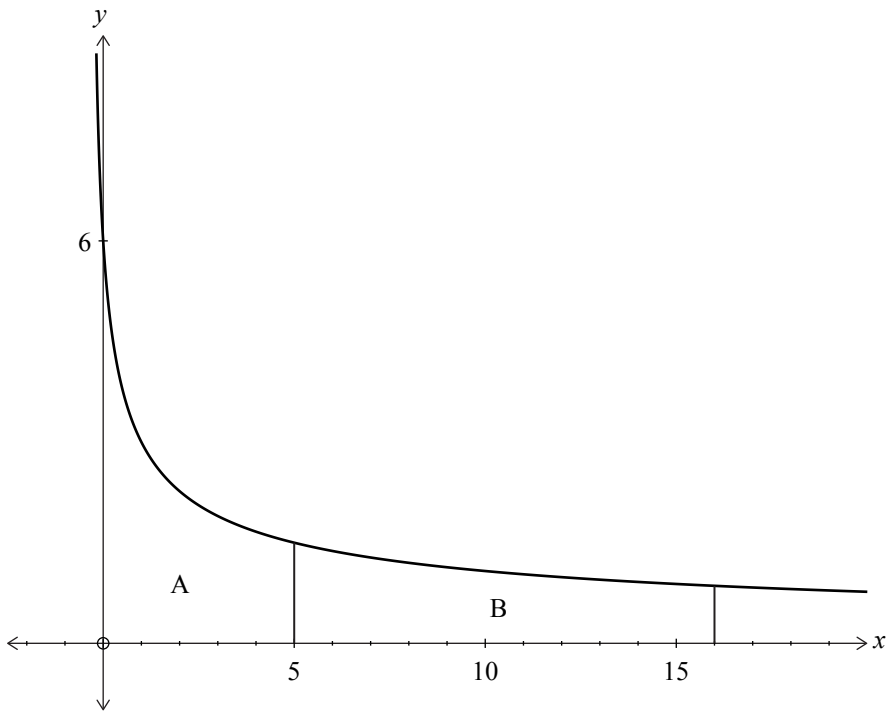
(d) The graph below shows the curve $y = \dfrac{6}{\sqrt{3x+1}}$.



Show that the areas of regions A and B are equal.

You must use calculus and show the results of any integration needed to solve the problem.

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(e) The rate of change of quantity N at any instant is given by the differential equation:

$$\frac{dN}{dt} = kN$$

If N has positive values N_1 and N_2 at times t_1 and $2t_1$ respectively, prove that

$$k = \frac{1}{t_1}\ln\left(\frac{N_2}{N_1}\right)$$

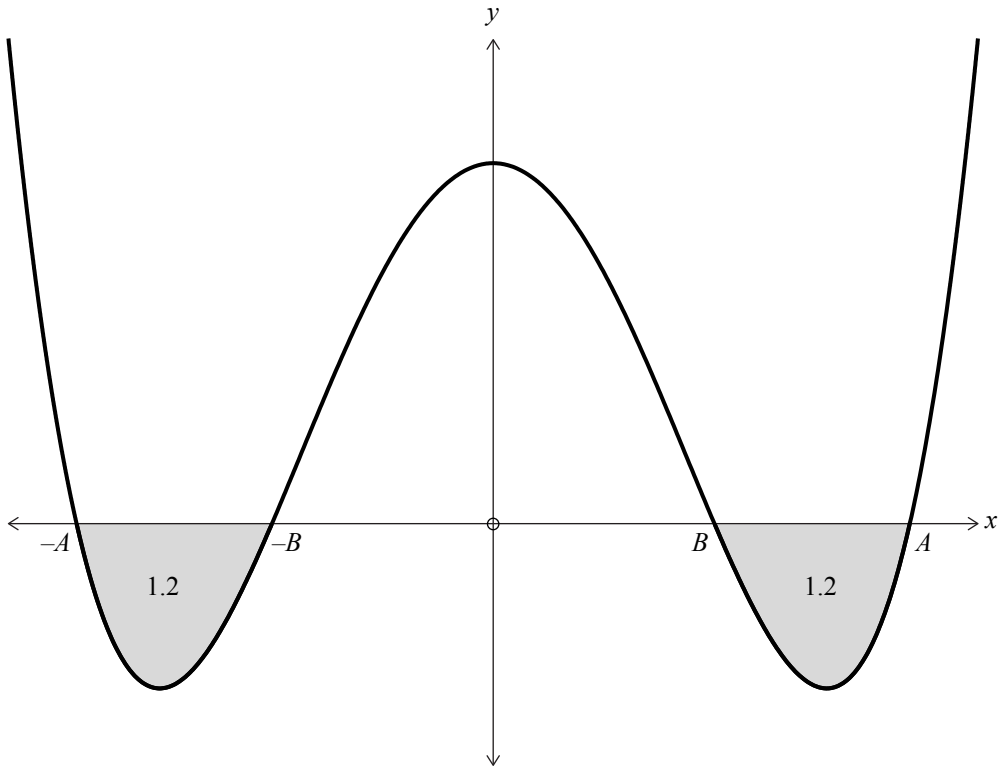
You must use calculus and show the results of any integration needed to solve the problem.

QUESTION TWO

ASSESSOR'S USE ONLY

(a) Find $\int \left(1+2e^{4x}\right) dx$.

(b) The graph of $y = f(x)$ shown below has the y-axis as a line of symmetry.
The areas of the shaded regions are shown.



If $\int_{-A}^{A} f(x)\,dx = 5.8$, what is the value of $\int_{-B}^{B} f(x)\,dx$?

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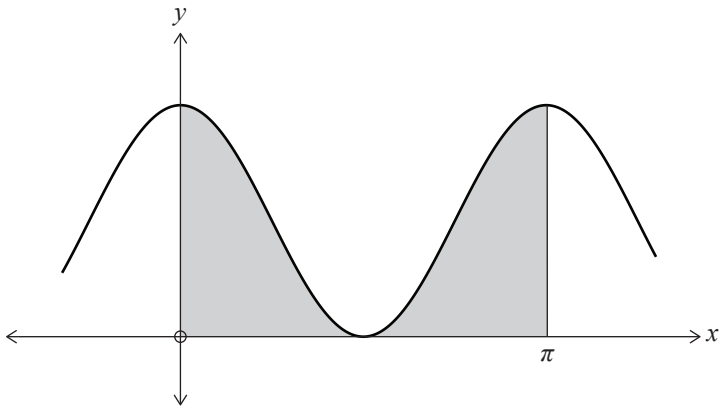
(c) Find k such that $\int_3^k \frac{8}{2x-5}\,dx = 10$.

You must use calculus and show the results of any integration needed to solve the problem.

(d) The diagram below shows the graph of the function $y = \cos^2 x$.



Find the area of the shaded region.

You must use calculus and show the results of any integration needed to solve the problem.

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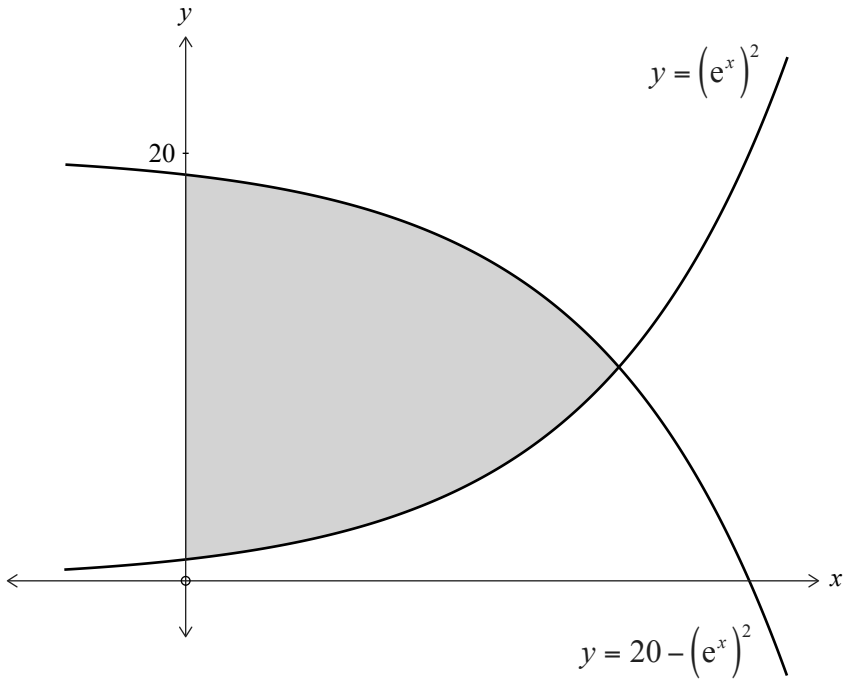
(e) The diagram below shows the graphs of the functions $y = \left(e^x\right)^2$ and $y = 20 - \left(e^x\right)^2$.



Find the area of the region shaded in the diagram.

You must use calculus and show the results of any integration needed to solve the problem.

QUESTION THREE

ASSESSOR'S USE ONLY

(a) Find $\int 24(2x-1)^3 \, dx$.

(b) Solve the differential equation $\frac{dy}{dx} = 4\sec^2 2x$, given that when $x = \frac{\pi}{8}$, $y = 5$.

(c) Find $\int_1^4 x + 1 + \frac{x}{x+1} \, dx$.

You must use calculus and show the results of any integration needed to solve the problem.

ASSESSOR'S USE ONLY

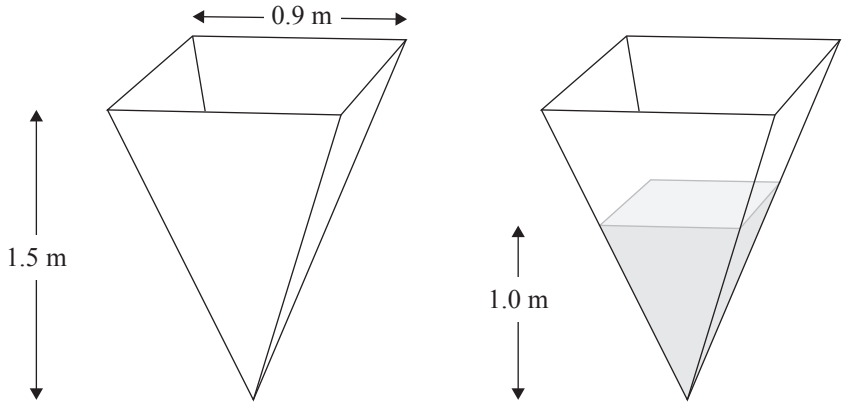
(d) If $\frac{dy}{dx} = \frac{4x}{4x^2 - 3} + \sqrt{x}$ and $y(1) = 2$, find $y(4)$.

Question Three continues on the following page.

ASSESSOR'S USE ONLY

(e) An inverted right pyramid has a square base and a height of 1.5 m.

The pyramid is initially filled with water to a depth of 1 m, as shown.



The energy required to pump water out of a tank of height H is given by:

$$E = 9800 \int_{H-d}^{H} (H-h)A(h)\,\mathrm{d}h$$

where E is the energy in joules
d is the initial depth of the water in the tank
h is the depth of the water in the tank at any instant
$A(h)$ is the area of the surface of the water at this instant.

Find the energy required to pump the water out of the tank shown.

You must use calculus and show the results of any integration needed to solve the problem.

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Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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