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Level 3 Calculus 2020

91579 Apply integration methods in solving problems

9.30 a.m. Monday 23 November 2020
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

(a) Find $\int \left(x + 2 + \frac{3}{x} \right) \mathrm{d}x$.

(b) For $t \geq 0$, the velocity of an object is given by $v(t) = 0.6\sqrt{t}$
where v is the velocity of the object in cm s^{-1}
and t is time in seconds from the start of the object's motion.

The object has a displacement of 5 cm at $t = 0$.

What will be the displacement of the object after 16 seconds?

(c) Find $\int_4^8 \frac{5x - 11}{x - 3} \, \mathrm{d}x$.

You must use calculus and show the results of any integration needed to solve the problem.

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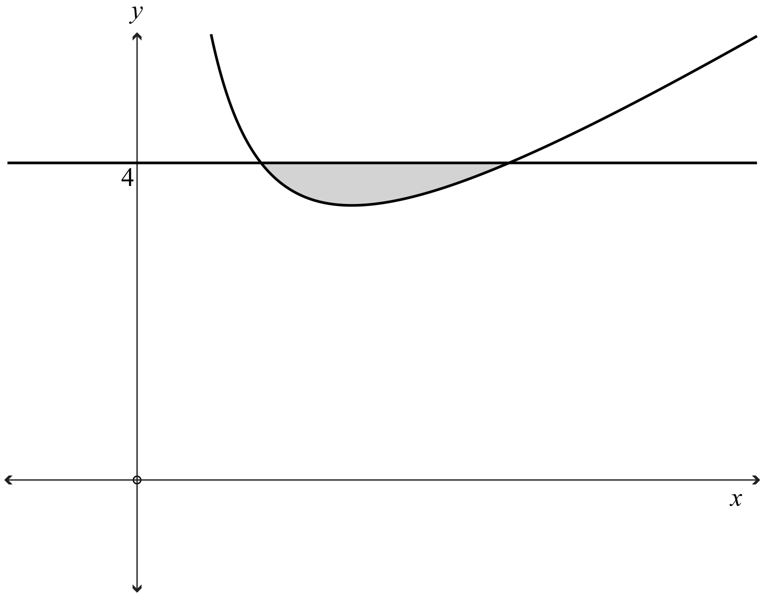
(d) The graph below shows the curve $y = x + \frac{3}{x}$ and the line $y = 4$.



Find the shaded area.

You must use calculus and show the results of any integration needed to solve the problem.

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(e) Consider the differential equation $\tan x \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sec^2 x}{y}, 0 < x < \frac{\pi}{2}$.

Given that $y = 2$ when $x = \frac{\pi}{4}$, find the value(s) of y when $x = \frac{\pi}{3}$.

You must use calculus and give the results of any integration needed to solve this problem.

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QUESTION TWO

(a) Find $\int \left(\pi - \dfrac{2}{x^2} \right) \mathrm{d}x$.

(b) Use the values given in the table below to find an approximation to $\int_0^3 f(x)\,\mathrm{d}x$, using Simpson's Rule.

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	1.1	1.8	2.1	2.4	2.7	1.8	1.3

(c) Find k such that $\int_1^k 9\sqrt{3x-2}\,\mathrm{d}x = 126$.

You must use calculus and give the results of any integration needed to solve this problem.

(d) If $\frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{y} \cdot \cos 4x$ and $y = 1$ when $x = \frac{\pi}{8}$, find the value of y when $x = \frac{\pi}{4}$.

You must use calculus and show the results of any integration needed to solve the problem.

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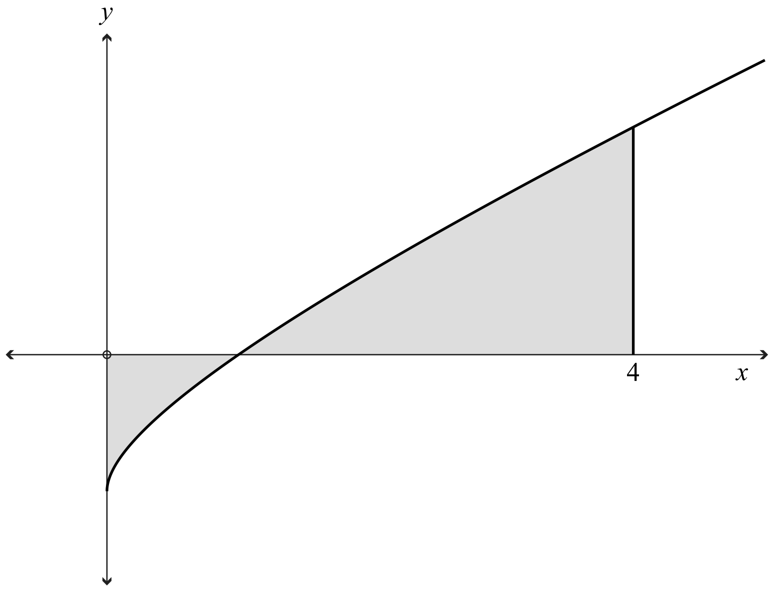
(e) The graph below shows the curve $y = x + 2\sqrt{x} - 3$.



Find the shaded area.

You must use calculus and give the results of any integration needed to solve this problem.

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QUESTION THREE

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(a) Find $\int \sec 2x \tan 2x \, \mathrm{d}x$.

(b) If $\frac{\mathrm{d}y}{\mathrm{d}x} = \cos 2x$ and $y = 1$ when $x = \frac{\pi}{12}$, find the value of y when $x = \frac{\pi}{4}$.

You must use calculus and give the results of any integration needed to solve this problem.

(c) An object originally moving at a constant velocity suddenly starts to accelerate. From the start of the object's acceleration the motion of the object can be modelled by the differential equation

$$\frac{dv}{dt} = t + e^{0.2t} \quad \text{for } 0 \leq t \leq 15$$

where v is the velocity of the object in m s^{-1}
and t is the time in seconds after the object starts to accelerate.

When $t = 0$, the velocity of the object was 8 m s^{-1}.

Find the velocity of the object when $t = 10$.

You must use calculus and give the results of any integration needed to solve this problem.

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Question Three continues on the following page.

ASSESSOR'S USE ONLY

(d) In radioactive decay, the rate at which the radioactive substance decays at any instant is proportional to the number of radioactive atoms present at that instant.

This can be modelled by the differential equation

$$\frac{\mathrm{d}N}{\mathrm{d}t} = kN$$

where N is the number of radioactive atoms present and t is the time in days.

A quantity of manganese-52 is produced.

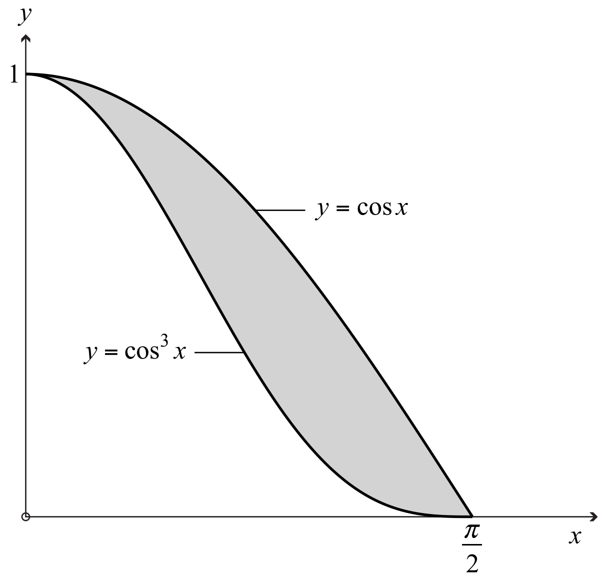
Manganese-52 is a radioactive isotope of manganese.

Manganese-52 has a half-life of 5.6 days (i.e. after 5.6 days, half of any atoms of manganese-52 would have decayed).

How long would it take for 95% of the manganese-52 to decay?

You must use calculus and give the results of any integration needed to solve this problem.

(e) The graph below shows the curves $y = \cos x$ and $y = \cos^3 x$ for $0 \le x \le \frac{\pi}{2}$.



Find the shaded area.

You must use calculus and give the results of any integration needed to solve this problem.

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Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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91579