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Level 3 Calculus 2021

91579 Apply integration methods in solving problems

Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (⧸⧹). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

(a)   Find $\int \left(\dfrac{x}{3} + \dfrac{3}{x} \right) dx$.

(b)   The gradient function of a curve is $\dfrac{dy}{dx} = \dfrac{8}{x^3}$.

    (i)   Find the equation of the curve if it passes through the point (1,3).

    You must use calculus and show the results of any integration needed to solve the problem.

    (ii)   Find the area enclosed by the curve, the x-axis, and the lines $x = 1$ and $x = 2$.

    You must use calculus and show the results of any integration needed to solve the problem.

(c)   An object's motion can be modelled by the differential equation $a(t) = 2 - \sin 2t$, where $t \geq 0$.

a is the acceleration of the object, in m s^{-2}

and t is time in seconds.

At $t = 0$, the object has a velocity of 1 m s^{-1} and a displacement of 3 m.

What is the displacement of the object at time $t = 5$?

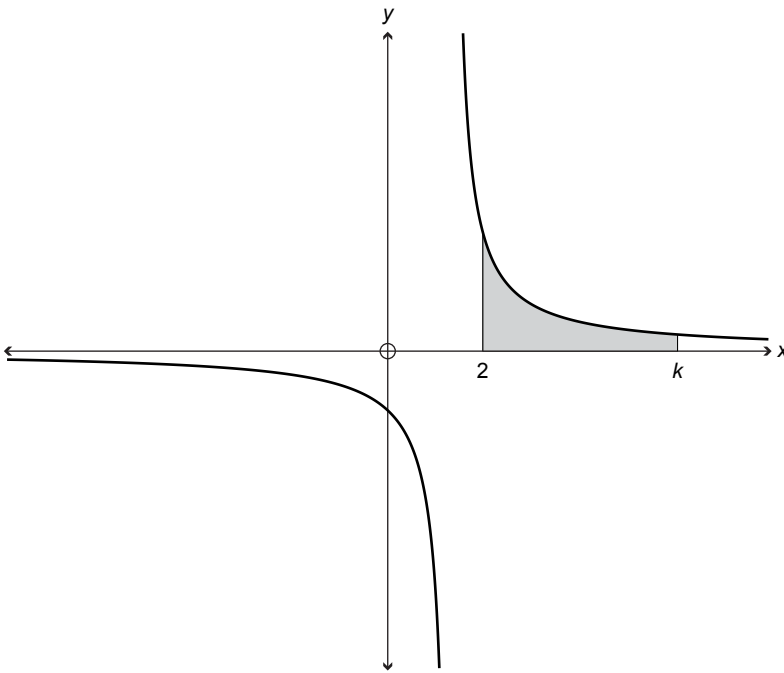*You must use calculus and show the results of any integration needed to solve the problem.*

(d) A water tank developed a leak.

6 hours after the tank started to leak, the volume of water in the tank was 400 litres.

10 hours after the tank started to leak, the volume of water in the tank was 256 litres.

The rate at which the water leaks out of the tank at any instant is proportional to the square root of the volume of the water in the tank at that instant.

How much water was in the tank at the instant it started to leak?

You must use calculus and show the results of any integration needed to solve the problem.

QUESTION TWO

(a) Find $\int \left(e^{4x} + 4\sqrt{x} \right) dx$.

(b) If $\int_{1}^{5} h(x)\,dx = 6$ what is the value of $\int_{1}^{5} \left(h(x) + 2 \right) dx$?

(c) Find $\int_{0}^{\frac{\pi}{8}} \sin 6x \, \sin 2x \, dx$.

You must use calculus and show the results of any integration needed to solve the problem.

(d) The diagram below shows part of the graph of the function $g(x) = \dfrac{6}{3x-4}$.
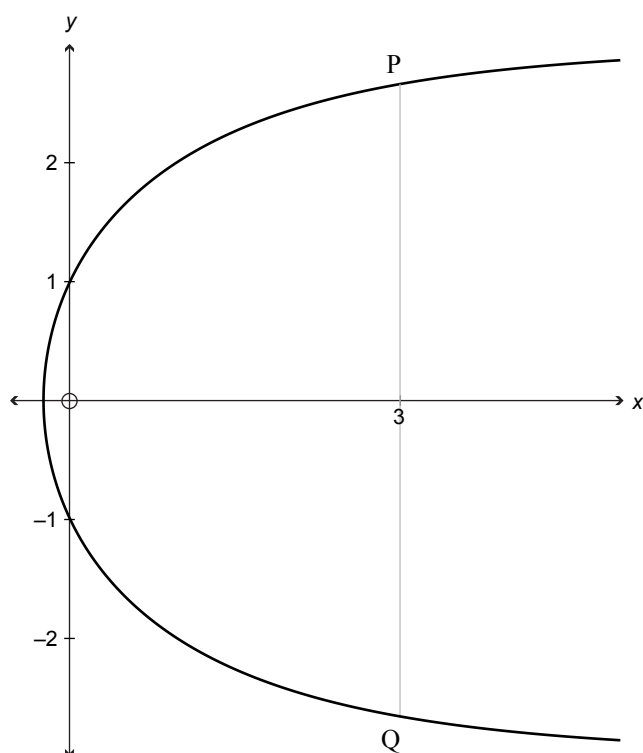

The area of the shaded region is 4.

Find the value of k.

You must use calculus and show the results of any integration needed to solve the problem.

(e) The diagram below shows the graph of a curve $y = f(x)$,

which satisfies the differential equation $\dfrac{dy}{dx} = \dfrac{2}{ye^{0.5x}}$.

Points P and Q are the points on the graph of the curve that have x-coordinates of 3.

What is the vertical distance between points P and Q?

You must use calculus and show the results of any integration needed to solve the problem.

QUESTION THREE

(a) Find $\int \left(x + \sqrt{x} \right)^2 dx$.

(b) Use the values given in the table below to find an approximation to $\int_1^{2.5} f(x)dx$ using the Trapezium Rule.
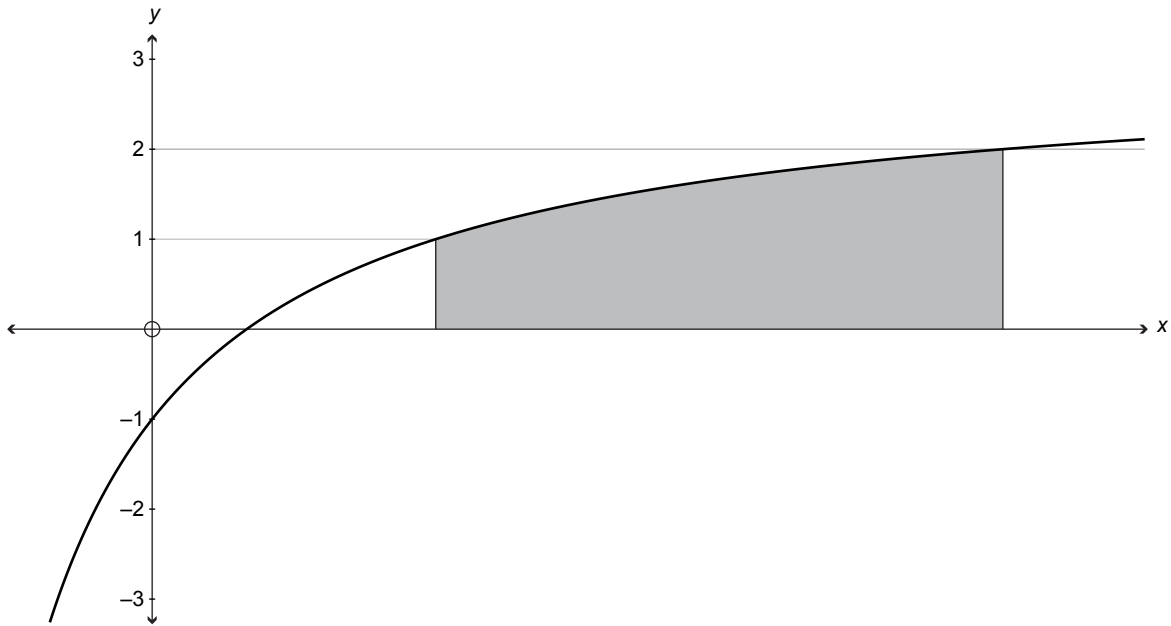
x	1	1.25	1.5	1.75	2	2.25	2.5
$f(x)$	0.8	1.1	1.5	1.9	2.2	2.1	2.4

(c) Consider the differential equation $\dfrac{dy}{dx} = \dfrac{\sec^2 2x}{y}$.

Given that $y = 2$ when $x = \dfrac{3\pi}{8}$, find the value(s) of y when $x = \pi$.

You must use calculus and show the results of any integration needed to solve the problem.

(d) The diagram below shows part of the graph of the function $y = \dfrac{3x-2}{x+2}$.
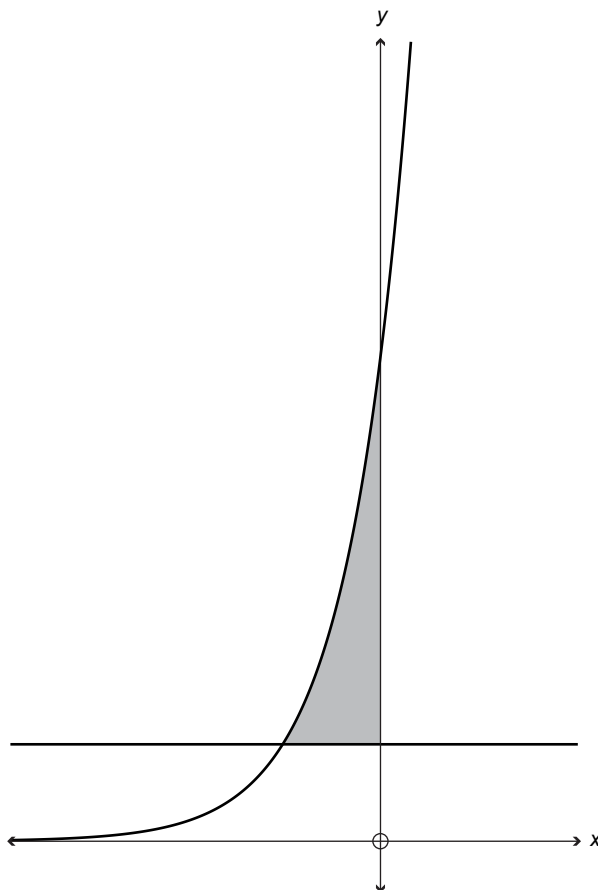

Find the shaded area.

You must use calculus and show the results of any integration needed to solve the problem.

(e)   The graph below shows the functions $y = \left(ke^x\right)^2$ and $y = k$, where k is a constant greater than 1.

Show that the shaded area is $\dfrac{k}{2}\left(k - 1 + \ln\dfrac{1}{k}\right)$.

You must use calculus and show the results of any integration needed to solve the problem.

Clearly show each step of your working.

QUESTION
NUMBER

QUESTION NUMBER	

Extra space if required.
Write the question number(s) if applicable.