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Level 3 Calculus 2022

91579 Apply integration methods in solving problems

Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2 – 12 in the correct order and that none of these pages is blank.

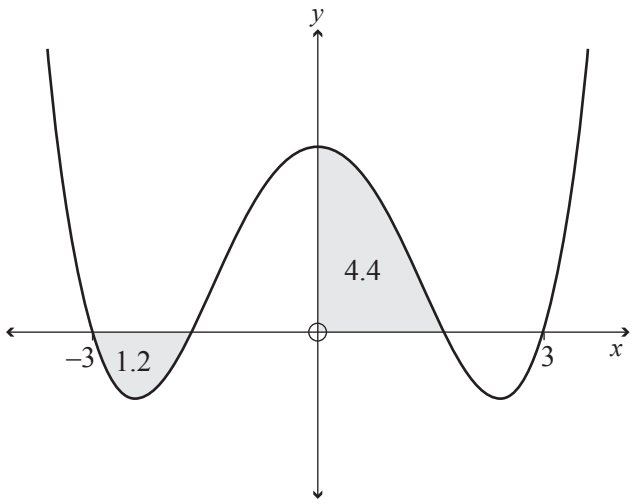
Do not write in any cross-hatched area (▨). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

(a) Find $\int \frac{4}{x} - \sec^2 x \, dx$.

(b) The graph of the function $y = f(x)$ below is symmetrical about the y-axis. The areas of the shaded regions are given.

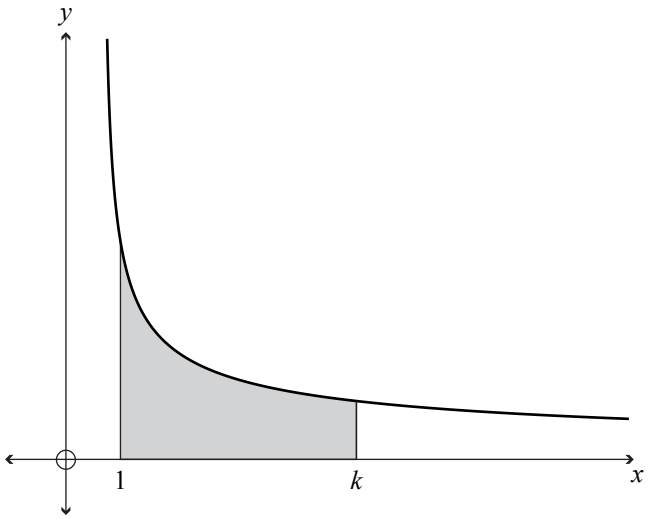


Find $\int_{-3}^{3} f(x) dx$.

(c) Find $\int_{0}^{\frac{\pi}{4}} \sin^2(2x) dx$.

You must use calculus and show the results of any integration needed to solve the problem.

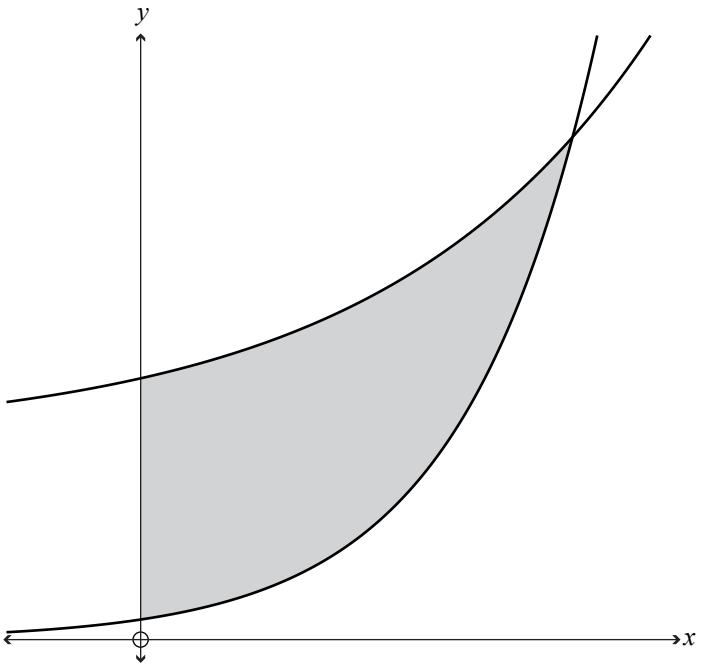
(d) The graph below shows part of the function $y = \dfrac{4}{\sqrt{3x-2}}$.



Find the value of k such that the shaded region has an area of 8.

You must use calculus and show the results of any integration needed to solve the problem.

(e) The graph below shows the functions $y = (e^x)^2$ and $y = 3e^x + 10$.



Find the exact value of the shaded area.

You must use calculus and show the results of any integration needed to solve the problem.

QUESTION TWO

(a) Find $\int \left(e^{3x} - \sqrt{x} \right) dx$.

(b) Find the value of k, given that $\int_1^k \frac{2}{\sqrt{x}}\, dx = 8$.

You must use calculus and show the results of any integration needed to solve the problem.

(c) Consider the differential equation $\frac{dy}{dx} = \frac{1}{3y^2(x-1)}$, where $x > 1$.

Given that $y = -1$ when $x = 2$, find the value(s) of x which give a y value of 1.

You must use calculus and show the results of any integration needed to solve the problem.

(d) An object's acceleration can be modelled by the equation $a(t) = 0.9e^{0.3t}$

where a is the acceleration of the object in m s^{-2}, and t is the time in seconds from the start of timing.

The object had a velocity of 10 m s^{-1} after 2 seconds.

How far did the object travel during the 5th second of its motion?

You must use calculus and show the results of any integration needed to solve the problem.

(e) A cylindrical tank of height 150 cm is originally full of oil.

The tank starts to leak out of a hole in its side.

The height h, in cm, of the oil left in the tank after it has been leaking for t minutes can be modelled by the differential equation $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{-1}{4}\sqrt{(h-6)^3}$.

Find how long it takes for the oil in the tank to be 15 cm above the bottom of the tank.

You must use calculus and show the results of any integration needed to solve the problem.

QUESTION THREE

(a) Find $\int (2x+5)^3 \, dx$.

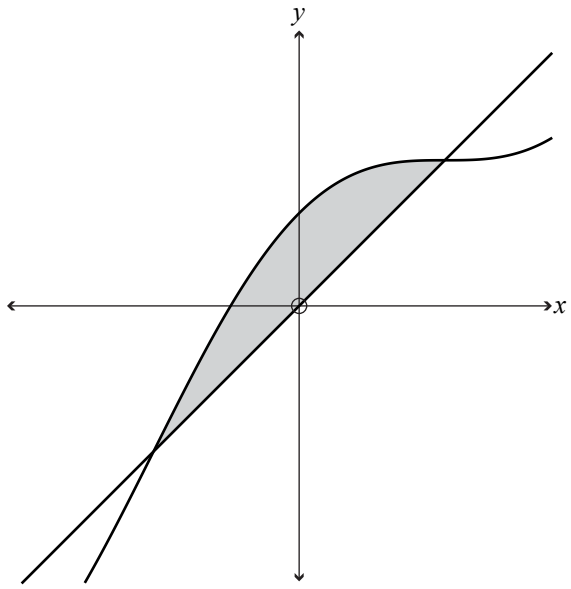
(b) Use the values given in the table below to find an approximation to $\int_0^2 f(x) dx$ using the Trapezium Rule.

x	0	0.4	0.8	1.2	1.6	2.0
$f(x)$	3.6	4.2	4.8	5.4	4.5	3.2

(c) Find $\int_5^8 \frac{4x-5}{x-3} dx$.

You must use calculus and show the results of any integration needed to solve the problem.

(d) The graph below shows part of the curve $y = x + \cos x$ and the line $y = x$.

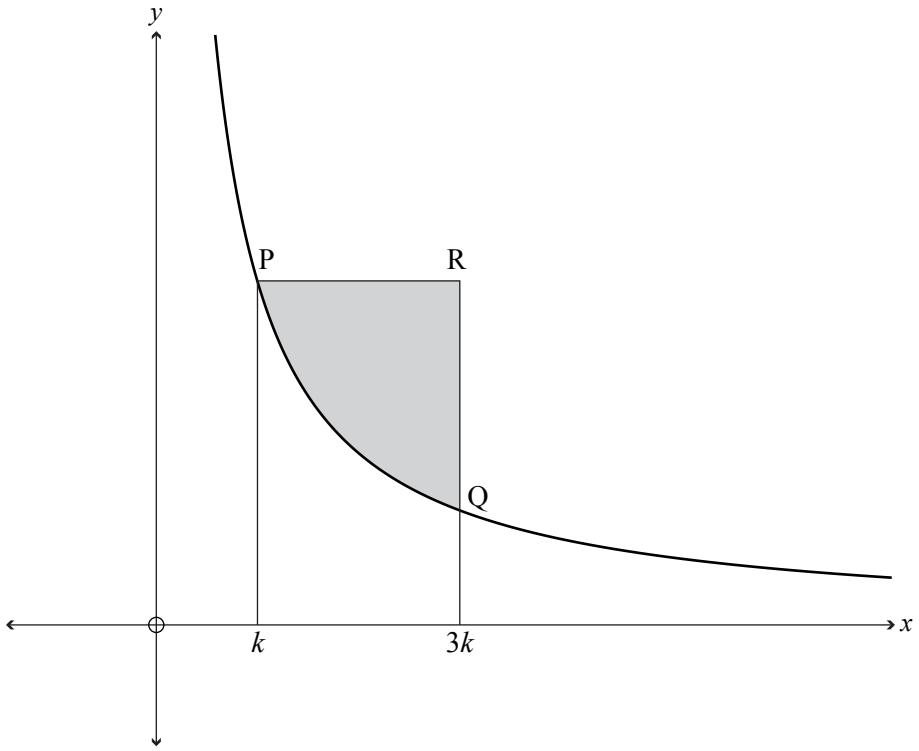


Find the shaded area.

You must use calculus and show the results of any integration needed to solve the problem.

Question Three continues on the next page.

(e) The graph below shows part of the curve given by the equation $y = \frac{2}{x}$.



Points P and Q lie on the curve with x-coordinates k and $3k$ respectively, where $k > 0$.

Point R is such that PR is parallel to the x-axis and QR is parallel to the y-axis.

The shaded area can be written in the form $a + b \ln c$, where a, b, and c are integers.

Find the values of a, b, and c.

You must use calculus and show the results of any integration needed to solve the problem.

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER