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NZQA | Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Level 3 Calculus 2023

91579 Apply integration methods in solving problems

Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (DO NOT WRITE IN THIS AREA). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

(a) Find $\int\left(3x+2+\frac{1}{3x+2}\right)\mathrm{d}x$.

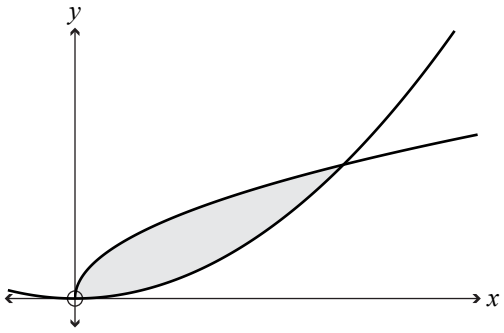
(b) An object's velocity can be modelled by the equation $v(t) = \sec^2 t$,
where v is the velocity of the object in km hr^{-1}, and t is the time in hours from the start of timing.

Initially the object was 3 km from a point P.

Find the distance of this object from the point P after $\frac{\pi}{4}$ hours.

You must use calculus and show the results of any integration needed to solve the problem.

(c) The graph below shows the functions $y = \sqrt{x}$ and $8y = x^2$.



Find the shaded area.

You must use calculus and show the results of any integration needed to solve the problem.

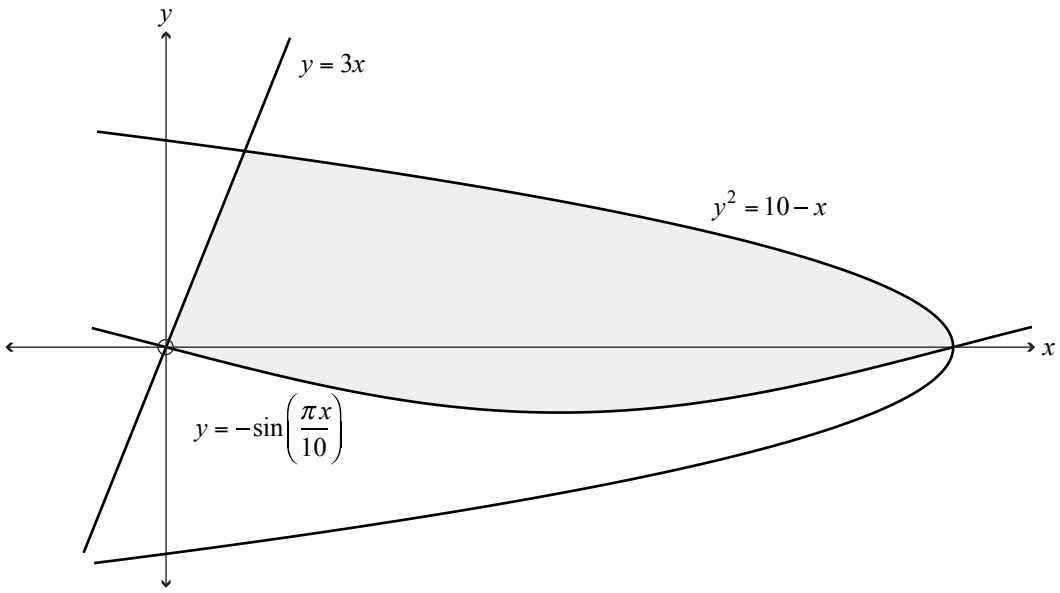
(d) Consider the differential equation $\frac{dy}{dx} = y\left(2x - 3x^2\right)$.

Given that $y = 1$ when $x = 2$, find the value(s) of y when $x = 1$.

You must use calculus and show the results of any integration needed to solve the problem.

(e) The shaded region in the diagram below is bounded by the three graphs:

$$y^2 = 10 - x$$
$$y = 3x$$
$$y = -\sin\left(\frac{\pi x}{10}\right)$$



Find the area of the shaded region.

You must use calculus and show the results of any integration needed to solve the problem.

QUESTION TWO

(a) Find $\int 4e^{2x-1}\,dx$.

(b) Solve the differential equation $\frac{dy}{dx} = (4x+1)^{-\frac{1}{2}}$, where $x \geq 0$, given that when $x = 6$, $y = 7.5$.

You must use calculus and show the results of any integration needed to solve the problem.

(c) Find the value of k, given that $\int_2^k \left(\frac{6x-3}{2x-3}\right) \mathrm{d}x = 3k$.

(d) Find $\int \frac{\cos 2x + \sin 2x}{\cos 2x - \sin 2x} \mathrm{d}x$.

(e) A cake factory has a container of liquid chocolate that is used in the manufacture of chocolate cakes.

The liquid chocolate is pumped out of the container so that the rate of change of the volume of liquid chocolate remaining in the container is proportional to the square of the volume of liquid chocolate remaining.

After one hour of use on a particular day, the volume of chocolate remaining is p litres, where p is a positive constant.

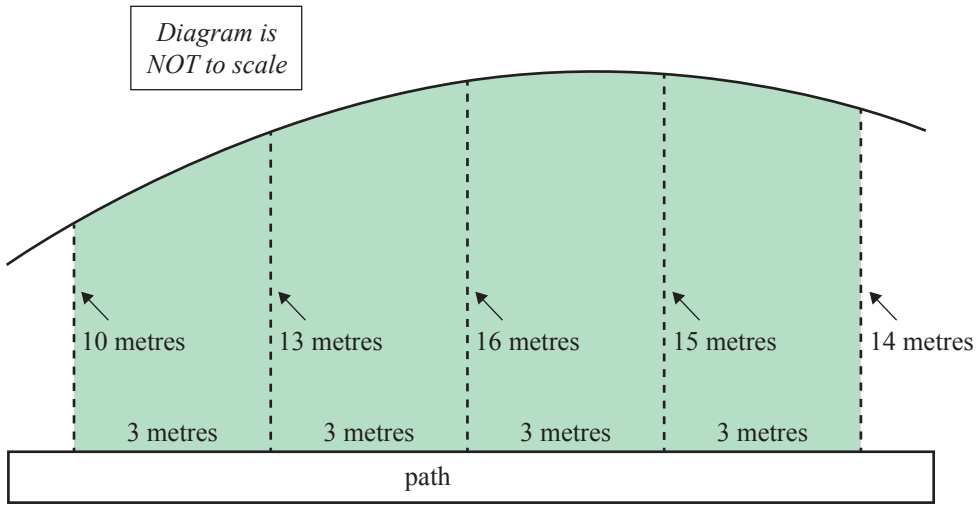
After a further one hour, there are only $\frac{4}{5}p$ litres of chocolate remaining in the container.

Write a differential equation that models this situation, and solve it to calculate how much liquid chocolate was in the container at the start of the day, giving your answer in terms of p.

QUESTION THREE

(a) A garden designer wants to find an approximation of the area of a section of a garden, shaded below.

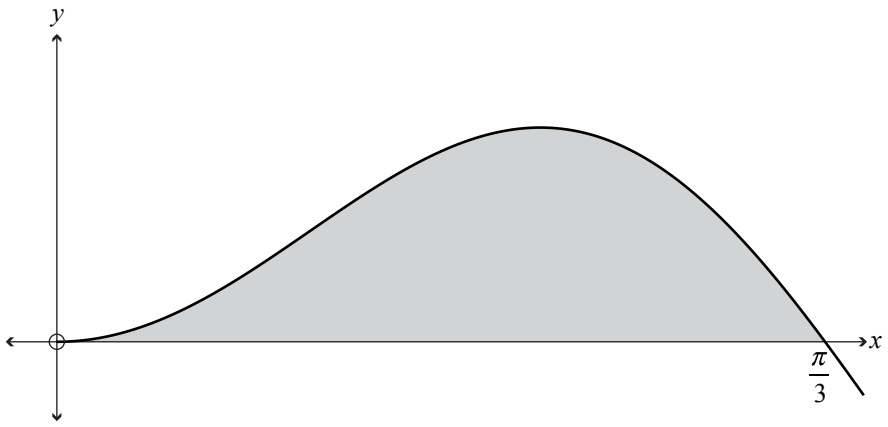
They take some measurements and these are shown in the diagram.



Using these measurements, and Simpson's Rule, find an approximation of the area of the garden section.

(b) Find $\int\left(\frac{\sqrt{x}-3}{\sqrt{x}}\right)\mathrm{d}x$.

(c) The graph below shows the function $y = 5\sin(3x)\sin(x)$.



Find the shaded area.

You must use calculus and show the results of any integration needed to solve the problem.

Question Three continues on the next page.

(d) An object's acceleration can be modelled by the equation $a = \dfrac{e^{2t}}{4e^{2t} - 3}$.

where $t \geq 0$ and a is the acceleration of the object in m s^{-2}
and t is the time, in seconds, from the start of timing.

At $t = 0$ seconds, the object had a velocity of 5 m s^{-1}.

Find the object's velocity when $t = 4$ seconds.

(e) Consider the differential equation $\left(1-x^2\right)\left(1+y\right)\dfrac{\mathrm{d}y}{\mathrm{d}x}+\left(1-x\right)\left(1-y^2\right)=0$.

Given that $y = 0$ when $x = 2$, find the value(s) of y when $x = 6$.

You must use calculus and show the results of any integration needed to solve the problem.

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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QUESTION NUMBER