Assessment Schedule – 2016

Mathematics and Statistics (Statistics): Apply probability concepts in solving problems (91585)

Evidence Statement

<table>
<tr><th>One</th><th>Expected Coverage</th><th>Achievement (u)</th><th>Merit (r)</th><th>Excellence (t)</th></tr>
<tr><td>(a)(i)</td><td><table><tr><th></th><th>Female shopper</th><th>Male shopper</th><th>Total</th></tr><tr><td>Tasted product</td><td>288</td><td>14</td><td>302</td></tr><tr><td>Did not taste product</td><td>240</td><td>17</td><td>257</td></tr><tr><td>Total</td><td>528</td><td>31</td><td>559</td></tr></table>P(tasted product) = $\frac{302}{559}$ = 0.54</td><td>Probability calculated.</td><td></td><td></td></tr>
<tr><td>(ii)</td><td>P(tasted product if female) = $\frac{288}{528}$ = 0.545
P(tasted product if male) = $\frac{14}{31}$ = 0.452
Relative risk = $\frac{0.545}{0.452}$ = 1.2
[1.2 times as likely to taste product if female compared to male.]</td><td>One conditional probability correctly calculated.</td><td>Conditional probabilities compared to determine how many times as likely.</td><td></td></tr>
<tr><td>(iii)</td><td>• Type 1: Only one sample
This claim is based on only one sample. It is likely that we would get different results if factors such as the location, the time or the demonstrator were changed.

• Type 2: Male sampling variability
Only a small number of male shoppers were approached, which means the probability estimate for tasting the product could be less accurate for them.</td><td>The fact that a generalisation is being made from the results of only one sample.
OR
Concern about the small amount of data used to form the male shopper estimate because of greater sampling variability.</td><td></td><td></td></tr>
</table>

(b)(i)	P(buy groceries on a weekend given regular shopping day) = 0.412 P(buy groceries on a weekend given no regular shopping day) = 0.289 The events "has a regular shopping day" and "buys groceries on a weekend" are not independent, since the probability of buying groceries on a weekend is different depending whether or not they have a regular shopping day. *Accept equivalent explanation.*	Full explanation given as to why the two events are not independent without further calculations being performed.		
(ii)	 P(buys groceries on a weekday) = $0.566 \times 0.588 + 0.434 \times 0.711 = 0.641$	Model formed and a useful probability found. OR consistent probability found from model with minor error.	Probability calculated.	
	P(regular shopping day and buys groceries on a weekend) = $0.566 \times 0.412 = 0.2332$ P(all three shoppers have a regular shopping day and buy groceries on a weekend) = $0.2332^3 = 0.01127$ • Type 1: Independence Assumption made that [the shopping habits and behaviours of] the three shoppers are independent. • Type 2: sampling without replacement Assumption made that the number of shoppers at this supermarket is sufficiently large that sampling without replacement is not required.	Probability correctly calculated for one shopper.	Probability correctly calculated for combined / joint probability (three shoppers).	Probability correctly calculated for combined / joint probability (three shoppers). AND One assumption stated clearly in context.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Reasonable start / attempt at one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t (with minor omission or error)	1 of t

<table>
<tr><th>Two</th><th>Expected Coverage</th><th>Achievement (u)</th><th>Merit (r)</th><th>Excellence (t)</th></tr>
<tr><td>(a)(i)</td><td>P(male) = 0.296
P(not stop to look at products) = 0.579
P(not stop to look at products and male) = 0.579 × 0.296 = 0.171 or 17.1%
The observer has assumed independence of the events “does not stop to look at the products” and “is male” to calculate the combined / joint probability of these two events.</td><td>Shows how the joint probability was calculated.</td><td>Shows how the joint probability was calculated.
AND
Explains the assumption of independence.</td><td></td></tr>
<tr><td>(ii)</td><td>P(stop to look at products) = 0.421
P(female) = 0.704
P(female and stop to look at products) = 0.387
Information used to construct two-way table.
<table>
<tr><th></th><th>Female</th><th>Male</th><th>Total</th></tr>
<tr><td>Stop</td><td>0.387</td><td></td><td>0.421</td></tr>
<tr><td>Not stop</td><td>0.317</td><td>0.262</td><td>0.579</td></tr>
<tr><td>Total</td><td>0.704</td><td>0.296</td><td>1</td></tr>
</table>
P(male and not stop to look at products) = 0.262
300 × 0.262 = 78.6
Predict 79 shoppers.</td><td>A significant step is made towards the solution e.g. model formed and made a useful calculation.</td><td>Prediction calculated.

Accept 78 shoppers.</td><td></td></tr>
<tr><td>(b)(i)</td><td>Possible reasons:
• The accuracy rate is based on a sample of observed customers (one in every ten).
• Each “observer” will have their own accuracy rate (the 86% rate is across all observers).
• The accuracy rate could also change over time as the “observer” gains more experience.
• The accuracy rate could be different depending on characteristics of the shopper being observed e.g. gender, age.
Accept other valid possible reasons.</td><td rowspan="2">One reason given for why the accuracy rate is only an estimate for the true probability.

OR

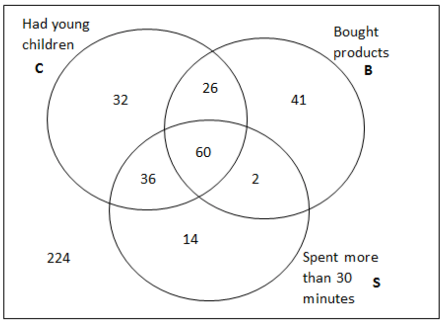
Some discussion how a simulation would allow the company to see that there is variability associated with estimates of accuracy rates.</td><td rowspan="2">One reason given for why the accuracy rate is only an estimate for the true probability.

AND

Some discussion how a simulation would allow the company to see that there is variability associated with estimates of accuracy rates.</td><td rowspan="2">One reason given for why the accuracy rate is only an estimate for the true probability.

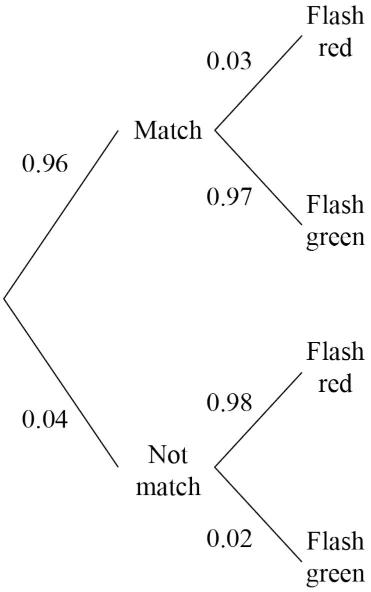
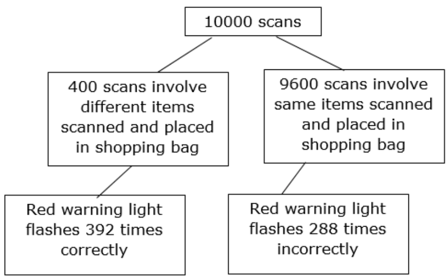
AND

A clear discussion of how a simulation would allow the company to see that they need to take into account sampling variation to make a decision on this observers accuracy rate.</td></tr>
<tr><td>(ii)</td><td>• A simulation would allow the company to see the variation in the number of correct age bands in samples of 42 shoppers.
• They could then compare what was observed (30 correct) to this simulated distribution to consider the likelihood of observed result happening, assuming the accuracy rate is 86%.</td></tr>
</table>

(c)	Information provided is used to construct an appropriate diagram (e.g. a Venn diagram) or to form statistical statements.  P(not C and not B and not S) $= \frac{224}{435} = 0.515$	A significant step is made towards the solution (at least three correct values in a constructed Venn diagram).	Model complete and value of 224 found.	Probability calculated.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Reasonable start / attempt at one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

Three	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)(i)	Possible reasons: • Type 1: Product error The item scanned may be different from the item put in the bag and weighed. • Type 2: Product weight variability Each item will vary in weight either side of what the weight is recorded as on the machine. [While the machine will have a tolerance, it may still not completely enclose all items e.g. ±1% of the recorded weight.] *Accept other valid possible reasons.*	One source of variation identified that would affect accuracy of the checking process.		

(a)(ii)	 P(item scanned is not the same as the item given the machine flashed red) $= \frac{\text{P(item scanned not same} \cap \text{P(flashed red)}}{\text{P(flashed red}}$ $= \frac{0.04 \times 0.98}{0.96 \times 0.03 + 0.04 \times 0.98} = \frac{0.0392}{0.0680} = 0.5765$ *Accept alternative but equivalent representations e.g. two-way table, or as shown below.* Suppose there were 10 000 scans at this supermarket.  Of the 680 scans where the red warning light flashes, 392 of these would be for scans that involve the item scanned not being the same as the item placed in the shopping bag. This gives a probability estimate of $\frac{392}{680} = 0.576$	Information provided is used to construct an appropriate diagram or to form statistical statements in an attempt to model the situation. A significant step is made towards the solution Eg. calculating the numerator.	Conditional probability calculated.	

(b)(i)	Using the model: $P(X = 4) = 0.076$ $P(X \leq 4) = 0.201$	Note: An error in the examination paper meant that Q3(b)(i) could not be answered. The error has been corrected in the pubished examination paper, and this answer has been amended to match. The maximum grade score awarded for Q3 was M5.
(b)(ii)	P(customers uses voucher no more than 9 days after receiving it) = 0.468 P(customers uses voucher no more than 8 days after receiving it) = 0.437 P(customers uses voucher no more than 7 days after receiving it) = 0.419 P(voucher used by expiry date) = $0.22 \times 0.468 + 0.35 \times 0.437 + 0.43 \times 0.419$ $= 0.436$	
(b)(iii)	Combining the model for the probability a customer uses the voucher after x days of receiving it and the model for how long it takes a customer to receive their voucher assumes independence, e.g. that it doesn't matter how long after the issue date the customer receives their voucher, the probability they will use the voucher after x days of receiving it stays the same. This may be an invalid assumption. Over half of the customers use their voucher 10 or more days after receiving the voucher, and it's likely the new requirement would change their behaviour, so the current model for the probability a customer uses the voucher after x days of receiving it would also change. The estimates given for how long it takes vouchers to be received (presumably by post) do not allow for vouchers to take more than three days to be received by customers. With mail delivery delays in some areas of NZ, the model should allow for the likelihood of longer delivery times. *Accept other reasonable limitations based on the model used in part (ii).*	

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Reasonable start / attempt at one part of the question.	1 of u	2 of u		1 of r			

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0–7	8–13	14–17	18–21