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Level 3 Mathematics and Statistics (Statistics) 2022

91585 Apply probability concepts in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability concepts in solving problems.	Apply probability concepts, using relational thinking, in solving problems.	Apply probability concepts, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2 – 12 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (▨). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

At a South Island high school, two option subjects were chosen by Year 10 students. The option subject choices of 167 Year 10 students were as follows:

- 62 had chosen to do a technology subject.
- 138 had not chosen to study a language subject.
- 10 had chosen a technology subject and a language.

(a) What proportion of students at this South Island high school had not chosen a technology subject, but were studying a language?

(b) Three individual students were chosen at random from the group of 167 Year 10 students.

Given all three were studying a technology subject, find the probability that all three were NOT studying a language.

(c) The school claims that if a student selects a technology subject, this has no effect on the probability of them also selecting a language subject.

Investigate the validity of this claim.

Use statistical reasoning to support your answer.

(d) The school is supporting their senior students (Years 11 to 13) to encourage them to pursue their preferred career pathway after high school. In total, the high school has 475 students in the senior school. A survey was conducted at the start of the academic year in 2022, with every senior student and their parents asked to identify the preferred pathway after high school.

In total, 71.6% of the students were intending on going to university, 63.4% of the total students were intending on going to university and were following their preferred career pathway, and 25.3% of the students were not intending on going to university, but were intending on following their preferred career pathway (e.g. starting an apprenticeship).

(i) Calculate the number of senior students who were not intending on going to university, and were not intending on following their preferred career pathway immediately after high school.

(ii) It is claimed that it is twice as likely for students to be following their preferred career pathway if they intended on going to university, compared to not going to university.

Comment on whether this data supports this claim.

Use calculations and statistical reasoning to support your answer.

(e) Nationally in New Zealand in 2022 there are 9985 students studying Level 3 Statistics for University Entrance. There are three possible examinations to take when studying Level 3 Statistics (91584, 91585, and 91586).

Over many years the following information is known about students entering Level 3 Statistics examinations

- 12% of students are entered in all three examinations.
- 9% of students are entered only for 91584.
- 14% students are entered only for 91585.
- 10% of students are entered only for 91586.
- 57% of students are entered for 91585 and 91586.
- 17% of students are entered for 91584 and 91585.
- 17% of students are entered for 91584 and 91586.

Given a student was entered for 91586, calculate the number of students in 2022 that were also entered for 91584.

QUESTION TWO

(a) The University of Auckland | Waipapa Taumata Rau is the largest university in New Zealand. The following table shows the number of male and female students attending the university from 2017 to 2020.

	2017	2018	2019	2020
Number of male students	18 168	18 192	18 296	18 065
Number of female students	24 134	24 525	24 746	24 760

(i) Which year, from 2017 to 2020, had the greatest proportion of male students attending Auckland University?

Support your answer with statistical calculation(s) and reasoning.

(ii) Provide TWO reasons why the proportions calculated from part (i) are only estimates of the true probability of a student being male at Auckland University in that year.

Reason One:

Reason Two:

(b) For every 50 first-year students at the University of Canterbury | Te Whare Wānanga o Waitaha in 2021, 9 took courses that did not have final examinations.

For students who took courses that had final examinations, 38% of students attended lectures regularly in person.

For students who took courses that did not have final examinations, 21% of students attended lectures regularly in person.

Let E be the event 'student took a course with a final examination' and A be the event 'student attended lectures regularly'.

(i) One first-year student from Canterbury University in 2021 is selected at random.

Calculate the probability that they attended lectures regularly.

(ii) A student believed that $P(E \cap A) + P(E' \cap A') = 1$.

Explain, using statistical reasoning, whether this student was correct.

(iii) Four first-year students in 2021 are chosen at random from Canterbury University.

Calculate the probability that three of the four students took a course without a final examination and attended lectures regularly.

Explain any assumptions made.

AND

Comment on one potential issue when applying the result of this calculation to all students taking first-year courses in 2022.

QUESTION THREE

(a) A standard New Zealand car number plate (non-personalised) looks like the following with a maximum of six characters.

https://upload.wikimedia.org/wikipedia/commons/8/8d/NEW_ZEALAND%2C_2002_-AAA-000_SERIES%2C_OLD_TALL_DIES%2C_LICENSE_PLATE_-_Flickr_-_woody1778a.jpg

The first three characters are letters that give an indication of when the car was first registered with Waka Kotahi | New Zealand Transport Agency (NZTA). The final three characters are numerical digits from 0 to 9.

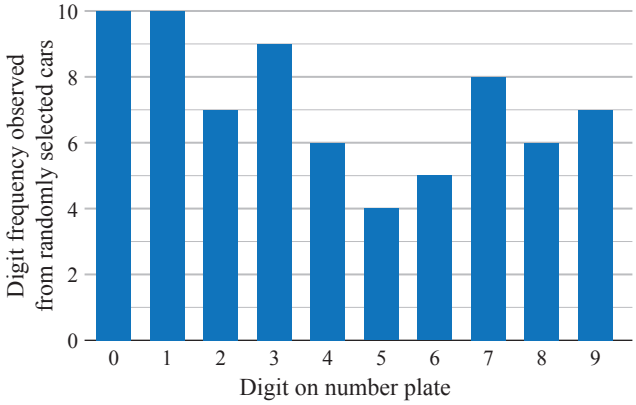
A Level 3 Statistics class were investigating whether the numerical digits on standard New Zealand number plates are equally likely to occur. They did this by looking at a website that sells cars across New Zealand, in every region of the country.

(i) The students had no prior knowledge of the distribution of the numbers on standard New Zealand car number plates.

What is the theoretical probability that they were most likely to give to each digit appearing?

Justify your answer with statistical reasoning.

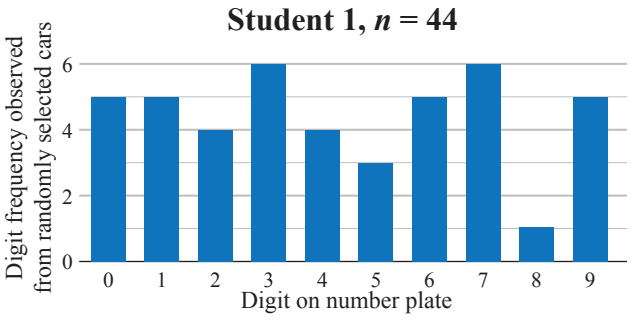
One student in the class produced a visualisation of their data, shown in the following bar chart.

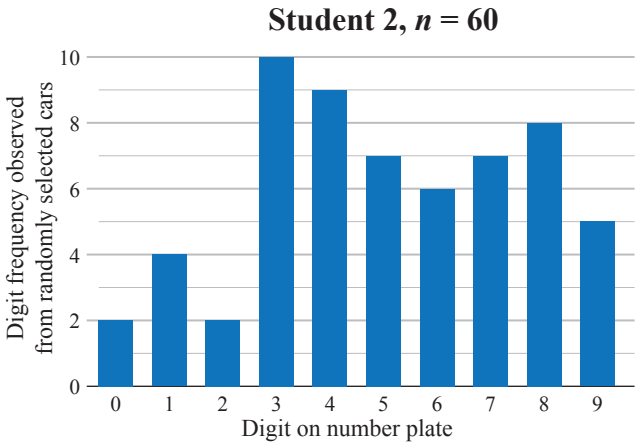


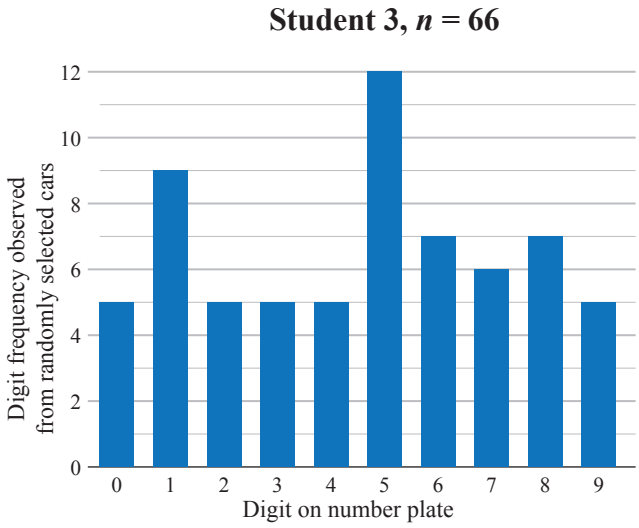
(ii) From the data given in the student's visualisation, calculate the probability a digit on the number plate was 5 or higher, given it was less than 8.

Question Three continues on the next page.

(b) Three other students' visualisations from their data collection are shown below, along with the total number of cars they collected data for (n).







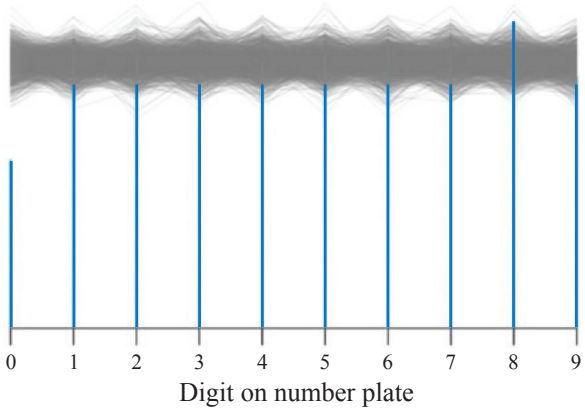
(i) Student 1 claimed that the other two students must have deliberately looked for number plates with the digit 8 on them.

Using statistical reasoning with calculations, evaluate the claim made by Student 1.

(ii) Explain, using statistical reasoning, why it is not possible to conclude from the results shown in the bar charts on pages 9 and 10 that digits on standard New Zealand number plates are not equally likely to occur.

(c) The teacher of the class grouped all the class's results together. They were then run through a simulation model 1000 times assuming that digits on standard New Zealand car number plates are equally likely to occur.

The total number of digits collected in the whole class was 2764. The diagram below shows the results of the simulation model. The blue vertical lines show the relative frequencies of each digit observed by the class. The grey band shows the variation expected for each digit, based on a total of 2764 digits and 1/10 probability for each digit.



Using statistical reasoning and the ideas about true probability, model probability, and experimental probability, what conclusions can be drawn about the assumption that numerical digits on standard New Zealand car number plates are equally likely to occur?

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER