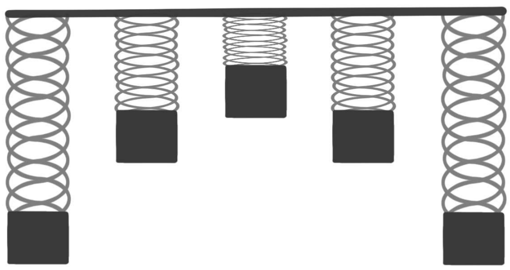
NCEA Level 3 Physics

Mechanical Systems

Study Notes



Site

YouTube

Mr Whibley

Mechanical Systems Cheatsheet

Force (N) Mass (kg)
$$F = ma$$
Acceleration (ms⁻²)

Momentum (kgms⁻¹) Velocity (ms⁻¹)
$$p = mv$$
Mass (kg)

Force (N) Duration (s)
$$\Delta p = F\Delta t$$
Momentum (kgms⁻¹)

Gravitational potential energy (J) Mass (kg)
$$\Delta E_p = mg\Delta h$$
Gravitational acceleration (ms⁻²) Height (m)

Work (J) Force (N) Distance (m)
$$W = Fd$$

Velocity (ms⁻¹)
$$E_{K(LIN)} = \frac{1}{2}mv^2$$
Linear kinetic energy (J) Mass (kg)

Distance to object 1 (m)
Distance to COM (m) Distance to object 2 (m)
$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$
Mass of object 1 (kg) Mass of object 2 (kg)

Distance (m)
Angle (rad)
$$d = r\theta$$
Radius (m)

Velocity (ms⁻¹)
Angular velocity (rads⁻¹)
$$v = r\omega$$
Radius (m)

Radius (m) Angular acceleration (rads⁻²)
$$a = r\alpha$$
acceleration (ms⁻²)

Change in angle (rad)
Angular velocity (rads⁻¹)
$$\omega = \frac{\Delta\theta}{\Delta t}$$
Change in time (s)

Change in angular velocity (rads⁻¹)
Angular acceleration (rads⁻²)
$$\alpha = \frac{\Delta\omega}{\Delta t}$$
Change in time (s)

Frequency (Hz)
$$\omega = 2\pi f$$
Angular velocity (rads⁻¹)

Frequency (Hz)
$$f = \frac{1}{T}$$
Period (s)

Rotational inertia (kgm²)
$$E_{K(ROT)} = \frac{1}{2}I\omega^2$$
Rotational kinetic energy (J) Angular velocity (rads⁻¹)

Initial angular velocity (rads⁻¹) Time (s)
$$\omega_f = \omega_i + \alpha t$$
Final angular velocity (rads⁻¹) Angular acceleration (rads⁻²)

Initial angular velocity (rads⁻¹)
Final angular velocity (rads⁻¹)
$$\theta = \frac{\omega_f + \omega_i}{2}t$$
Angle (rad) Time (s)

Initial angular velocity (rads⁻¹) Time (s)
$$\omega_f^2 = \omega_i^2 + 2\alpha t$$
Final angular velocity (rads⁻¹) Angular acceleration (rads⁻²)

Angle (rad) Angular acceleration (rads⁻²)
$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$
Initial angular velocity (rads⁻¹) Time (s)

Angular acceleration (rads⁻²)
$$\tau = I\alpha$$
Torque (Nm⁻¹) Rotational inertia (kgm²)

Force (N)
$$\tau = Fr$$
Torque (Nm⁻¹) Radius (m)

Velocity (ms⁻¹) Radius (m)
$$L = mvr$$
Angular momentum (kgms⁻¹) Mass (kg)

Angle (rad) Angular acceleration (rads⁻²)
$$\theta = \omega_f t - \frac{1}{2}\alpha t^2$$
Initial angular velocity (rads⁻¹) Time (s)

Gravitational constant (6.67×10^{-11} Nm²kg⁻²) Mass 1 (kg) Mass 2 (kg)
$$F_g = \frac{GMm}{r^2}$$
Gravitational force (N) Radius (m)

Mass (kg) Velocity (ms⁻¹)
$$F_c = \frac{mv^2}{r}$$
Centripetal force (N) Radius (m)

Rotational inertia (kgm²)
Angular velocity (rads⁻¹)
$$L = I\omega$$
Angular momentum (kgms⁻¹)

Stiffness (Nm⁻¹) Displacement (m)
$$F = -ky$$
Spring force (N)

Elastic potential energy (J)
$$E_p = \frac{1}{2}ky^2$$
Stiffness (Nm⁻¹) Displacement (m)

Period (s) Length (m)
$$T = 2\pi\sqrt{\frac{l}{g}}$$
Gravitational acceleration (ms⁻²)

Period (s) Mass (kg)
$$T = 2\pi\sqrt{\frac{m}{k}}$$
Stiffness (Nm⁻¹)

$$y = A\sin\omega t$$
Displacement (m) Amplitude (m) Time (s) Angular velocity (rads⁻¹) Time (s)
$$y = A\cos\omega t$$

$$v = A\omega\cos\omega t$$
Velocity (ms⁻¹) Amplitude (m) Angular velocity (rads⁻¹) Time (s)
$$v = -A\omega\sin\omega t$$

$$a = -A\omega^2\sin\omega t$$
acceleration (ms⁻²) Amplitude (m) Angular velocity (rads⁻¹) Time (s)
$$a = -A\omega^2\cos\omega t$$

Angular velocity (rads⁻¹)
$$a = -\omega^2 y$$
Acceleration (ms⁻²) Displacement (m)

Mr Whibley

Speed

Speed describes how quickly an object's position changes.

The vector of speed is velocity.

Its scientific units are...

metres per second

m/s or ms^{-1}

We can describe speed with the equation...

distance (m)

$$v = \frac{d}{t}$$

Speed (ms^{-1}) time (s

Example: How long does it take for a car moving at $10ms^{-1}$ to travel $25m$?

$$v = \frac{d}{t}$$

$$t = \frac{d}{v} = \frac{25}{10} = 2.5s$$

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Acceleration

Acceleration describes how quickly an object's speed changes.

Its scientific units are...

metres per second, per second.

$m/s/s$ or ms^{-2}

We describe acceleration with the equation...

Speed (ms^{-1})

$$a = \frac{v}{t}$$

Acceleration (ms^{-2}) time (s

When an object accelerates against its velocity we say it decelerates.

Example: A car accelerates from rest at $5ms^{-2}$ for 7 seconds.
Calculate it's new speed.

$$a = \frac{v}{t}$$

$$v = at = 5ms^{-2} \times 7s = 35ms^{-1}$$

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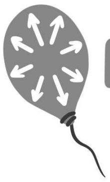
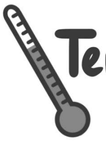
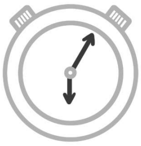
Scalars and vectors

Vectors are quantities with size and direction.
Scalars only have size.

Scalars


Pressure

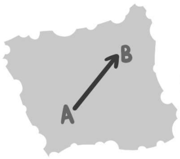

Temperature


Time

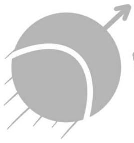
Mass

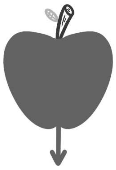

Energy

Vectors

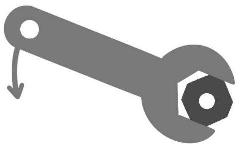


Displacement


Velocity


Acceleration


Force


Torque

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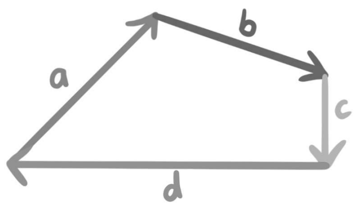
Equating Vectors

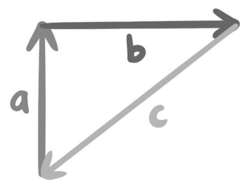
Similar to scalars, vectors form equations. To do this we can use these techniques...

Circles are zeroes

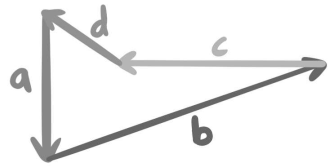
Vectors forming closed paths equal zero.



$a + b + c + d = 0$



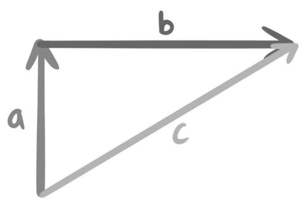
$a + b + c = 0$



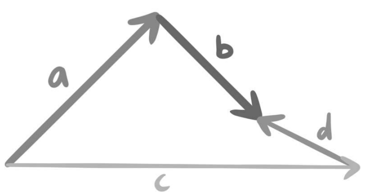
$a + b + c + d = 0$

Equivalent paths

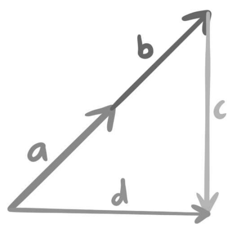
Vectors or sets of vectors that start and finish in the same place are equal.



$a + b = c$



$a + b = c + d$



$a + b + c = d$

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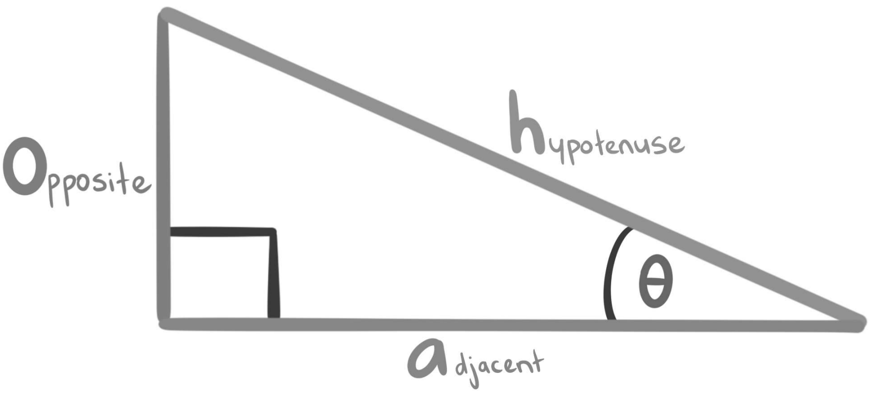
Trigonometry refresher



* Pythagoras

$$h^2 = o^2 + a^2$$

$h = \sqrt{o^2 + a^2}$

$o = \sqrt{h^2 - a^2}$

$a = \sqrt{h^2 - o^2}$

* Soh-Cah-Toa

$\sin\theta = \frac{o}{h}$ $\quad$ $\cos\theta = \frac{a}{h}$ $\quad$ $\tan\theta = \frac{o}{a}$

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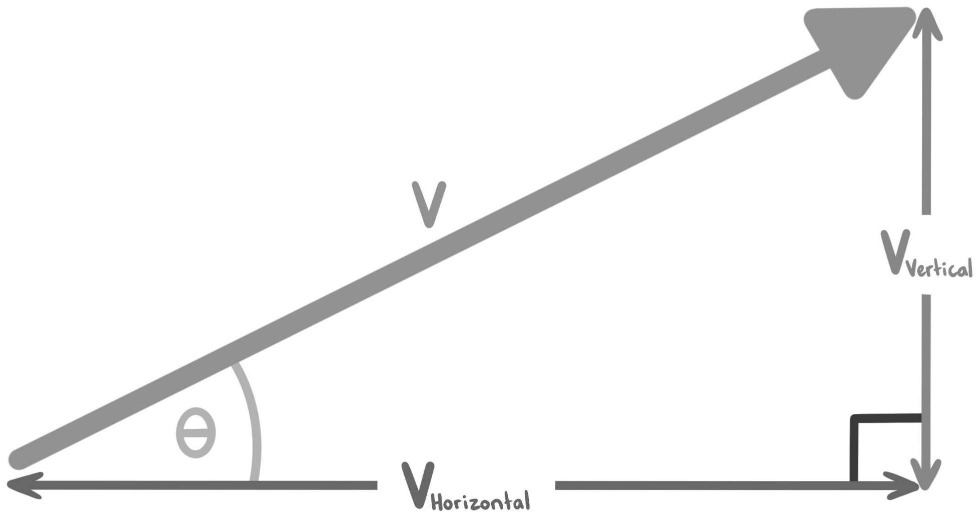
Vector Components

It is often useful to separate a vector into horizontal and vertical components.

Knowing the total magnitude and the angle we can use SohCahToa.



$V_{Horizontal} = V\cos\theta$ $\qquad$ $V_{Vertical} = V\sin\theta$

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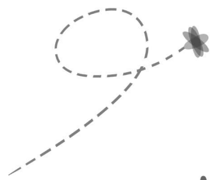
Momentum

Momentum is a measurement of motion that takes into account both mass and velocity.

It has units of $kgms^{-1}$.

$$\underbrace{p}_{\text{Momentum } (kgms^{-1})} = \underbrace{m}_{\text{Mass } (kg)}\ \underbrace{v}_{\text{Velocity } (ms^{-1})}$$

A bee and a car might travel at the same velocity, but have very different momentums due to their considerable mass difference.



$$p = 0.001kg \times 10ms^{-1} = 0.01kgms^{-1}$$

$$p = 2000kg \times 10ms^{-1} = 20\,000kgms^{-1}$$

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Impulse

The longer a collision takes, the smaller the force.

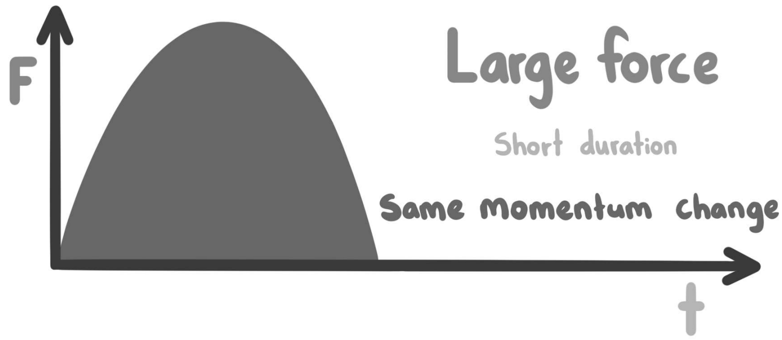
We call this concept impulse.

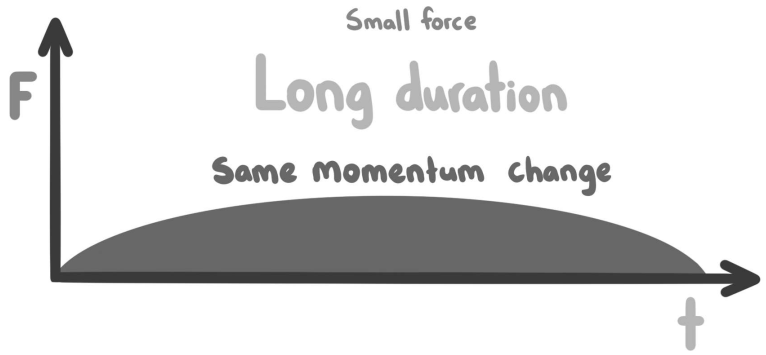
Force (N)

$$F = \frac{\Delta p}{\Delta t}$$

Momentum change ($kgms^{-1}$)

Duration (s)





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Conservation of momentum

The total momentum of a set of objects will only change if an external force acts upon them.

This is called conservation of momentum.

Before

$20 + -30 = -10$

After

$-25 + 15 = -10$

Billiard balls experience gravity and support force. Though they are external, they are balanced. The net external force is therefore zero and momentum is conserved.

A ball in flight experiences an unbalanced external force due to gravity. Consequently momentum is not conserved.

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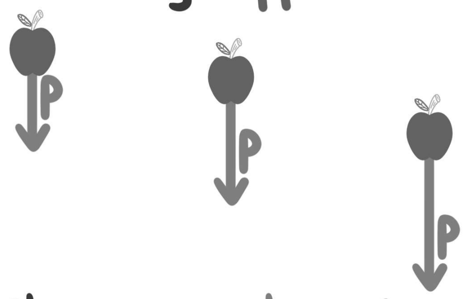
What is an external force?

Consider a falling apple...



As it falls its momentum increases. We could say that momentum isn't conserved because gravity is an external force.

An external force is a force applied by an object whose momentum we've ignored. Often because it's impractical or impossible to measure.

In the case of the apple, we've ignored the equal and opposite momentum the apple's gravity has imparted to the Earth!

Momentum appears to not be conserved only when part of a system is ignored.

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2D momentum example

Determine the speed of Ball B after the collision.

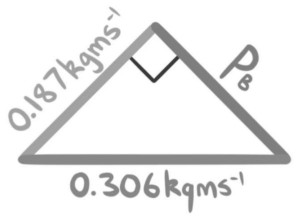
Determine total initial momentum.

$$P_{Initial} = m_A V_{Ai} = 0.170\,kg \times 1.80\,ms^{-1} = 0.306\,kgms^{-1}$$

Determine final Ball A momentum.

$$P_A = m_A V_{Af} = 0.170\,kg \times 1.10\,ms^{-1} = 0.187\,kgms^{-1}$$

Determine final Ball B momentum.



$$P_B = \sqrt{0.306^2 - 0.187^2}$$
$$= 0.242\,kgms^{-1}$$

Determine final Ball B velocity.

$$V_B = \frac{P_B}{m_B} = \frac{0.242\,kgms^{-1}}{0.120\,kg} = 2.02\,ms^{-1}$$

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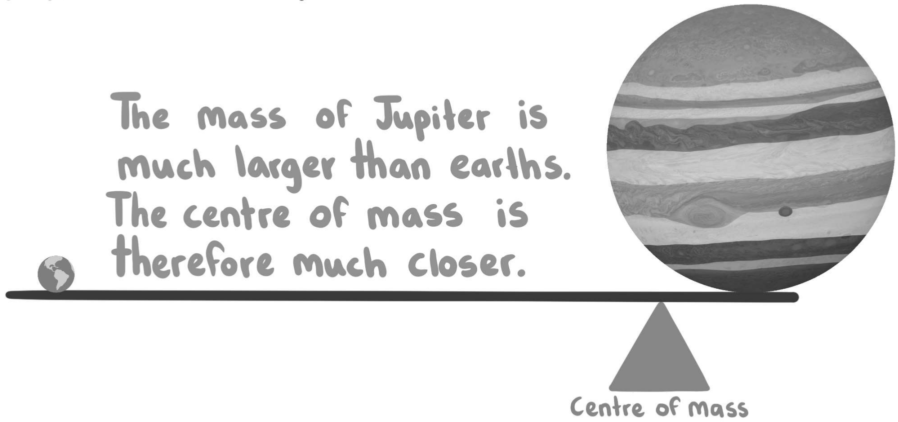
Centre of mass

Centre of mass describes the average position of a set of objects, weighted by their masses.

Two objects on a pivot will balance at their centre of mass.

The mass of Jupiter is much larger than earths. The centre of mass is therefore much closer.



Distance to Centre of mass (m)

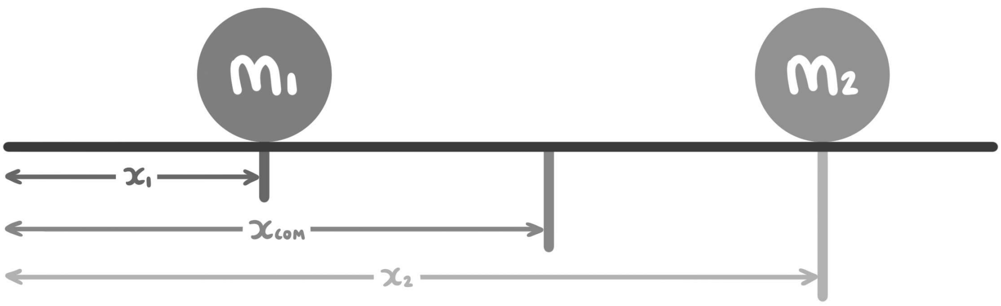
Distance 1 (m)

Distance 2 (m)

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

mass 1 (kg)

mass 2 (kg)



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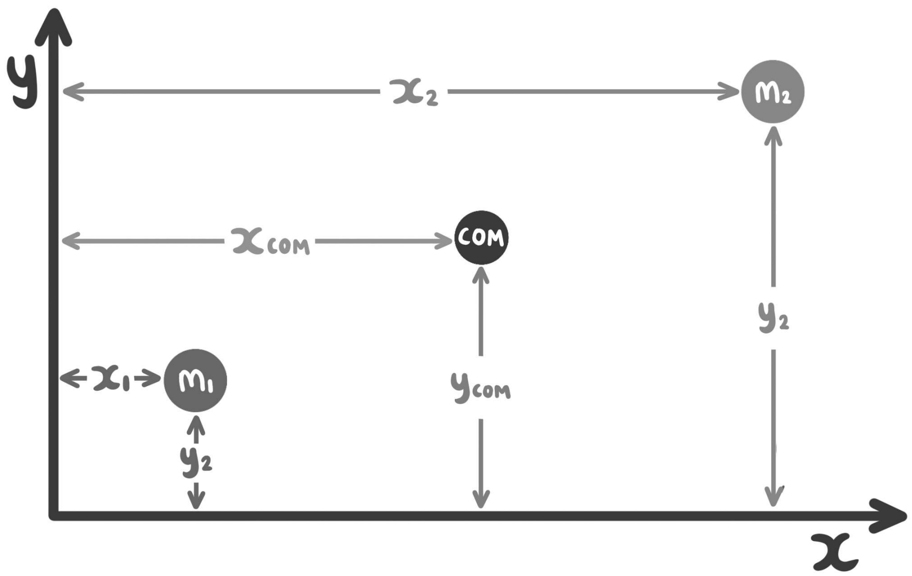
Centre of mass 2D

Finding the centre of mass of objects in 2D is done by finding x_{com} and y_{com}.



$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$y_{com} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

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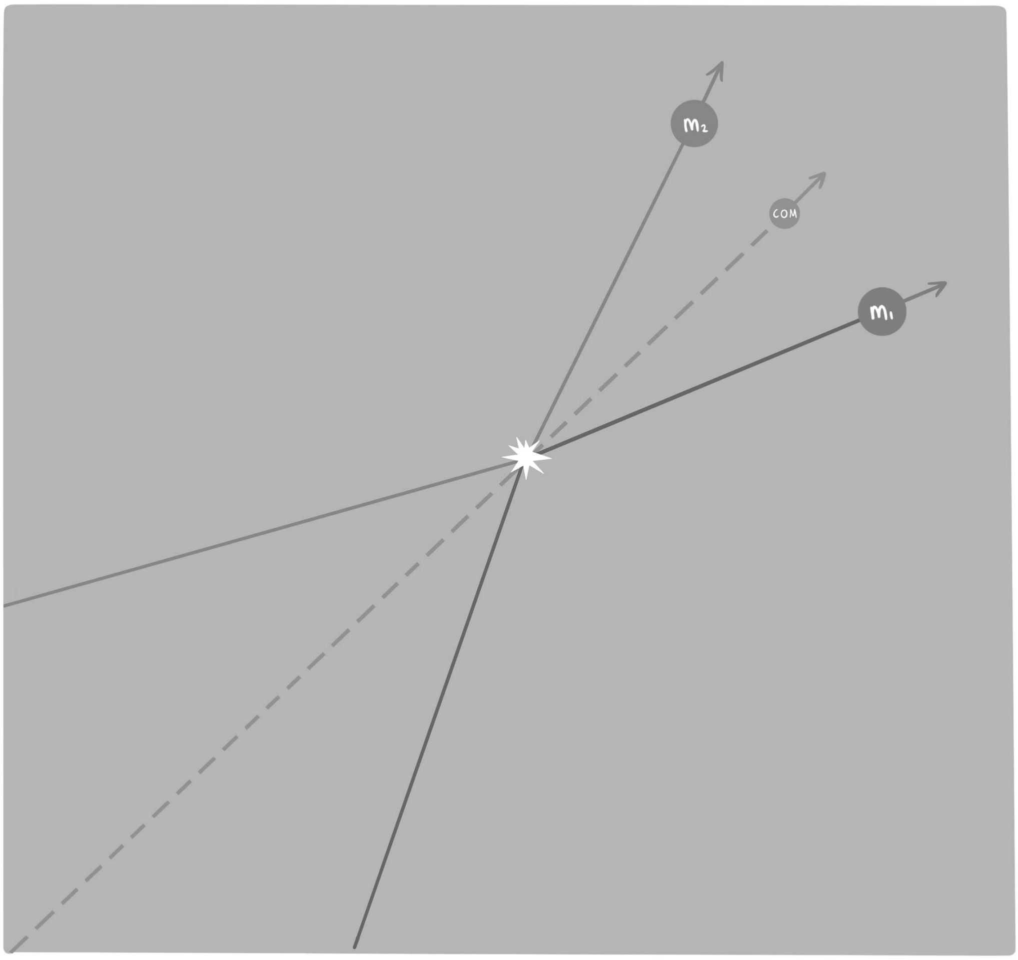
Centre of Mass Collisions

Provided there is no resultant external force, the motion of the centre of mass will not change.



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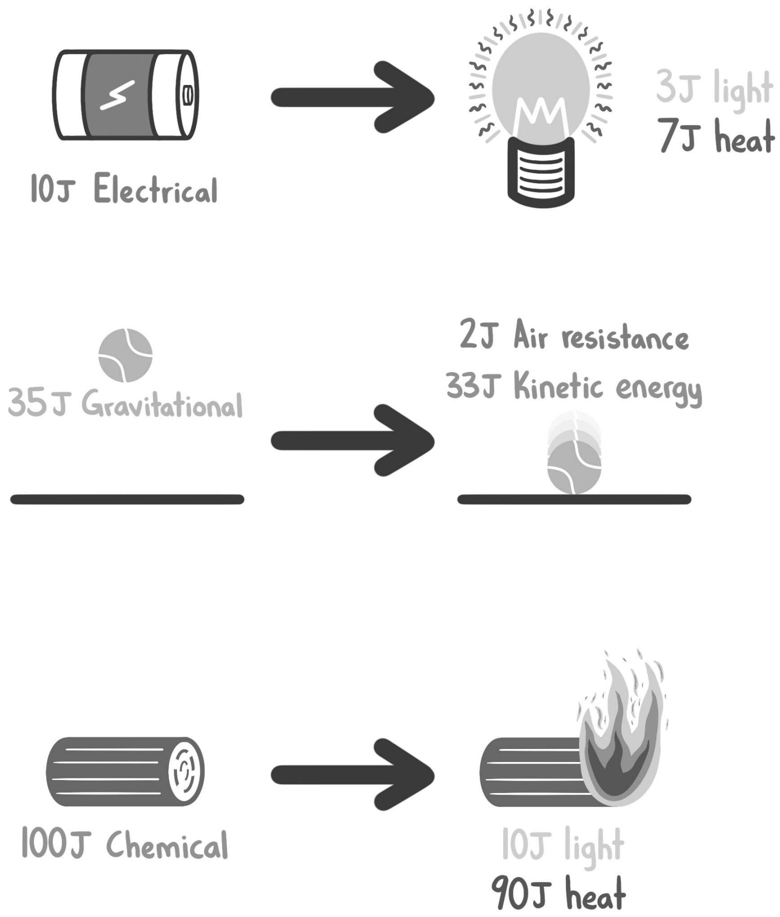
Conservation of energy

Energy is never created or destroyed, only transferred into different forms.

This idea is called Conservation of energy.



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Work

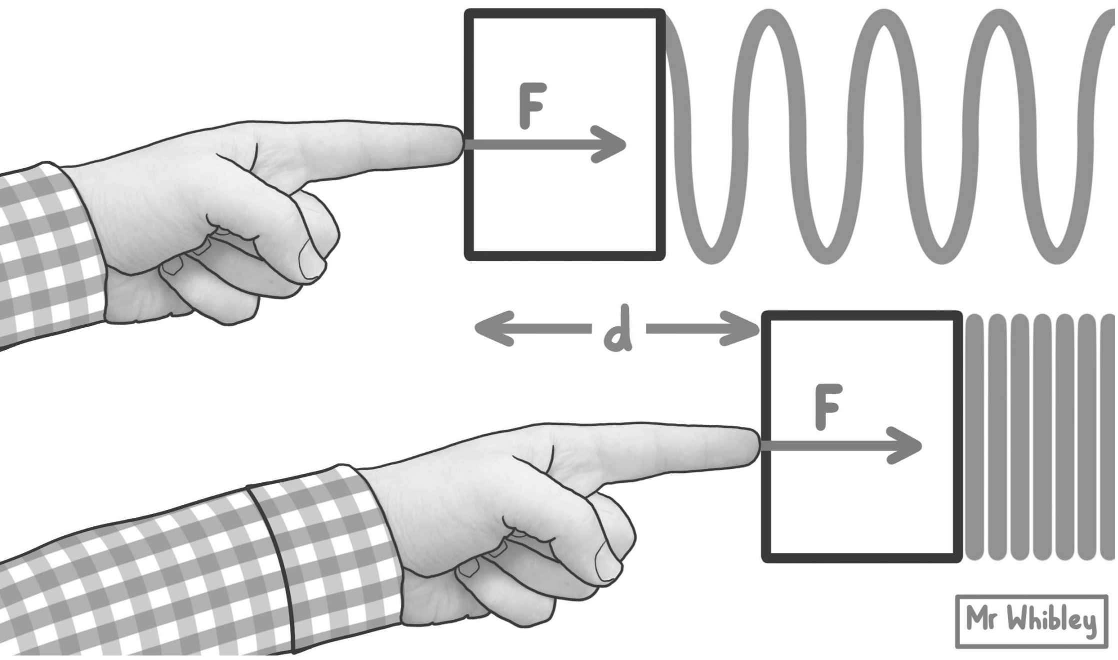
When energy has been used, we say that work has been performed.

Work is essentially another term for energy, it has the same units Joules (J).

When a force is performed over a distance the work performed is...

$$W = Fd$$

Work (J), Force (N), Distance (m)



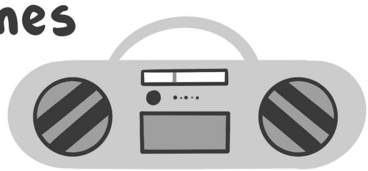
Power

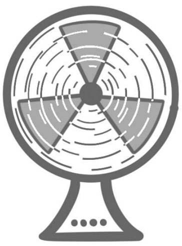
Power is the rate of energy consumption.

It is measured in Joules per second, which are commonly called Watts (W).

$$\underset{\text{Power (W)}}{P} = \frac{\overset{\text{Energy (J)}}{E}}{\underset{\text{Time (s)}}{\Delta t}}$$

A 25W stereo consumes 25J per second.





A 10W fan consumes 10J per second.

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Force

A force is an influence that acts to accelerate objects.

It is measured in Newtons (N).

Newton's 1st law

An object will only change its motion (velocity) if a force acts upon it.

Newton's 2nd law

When a force acts upon an object it will accelerate proportional to its mass.

Force (N) → $F = ma$ ← Acceleration (ms^{-2})

Mass (kg)

Newton's 3rd law

Forces exist in equal and opposite pairs.

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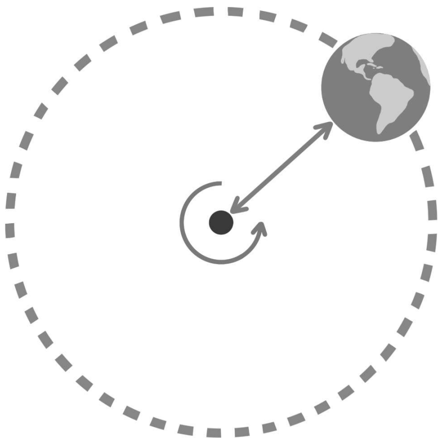
Circular motion

Objects in circular motion are described using the following terms...



Radius (r) half circle width

Diameter (2r) Circle width

Circumference ($2\pi r$) Length of one lap

Revolution Complete lap

Period (T) Seconds per revolution

Frequency (f) Revolutions per second

$f = \frac{1}{T}$ $T = \frac{1}{f}$ $v = \frac{2\pi r}{T}$

Velocity

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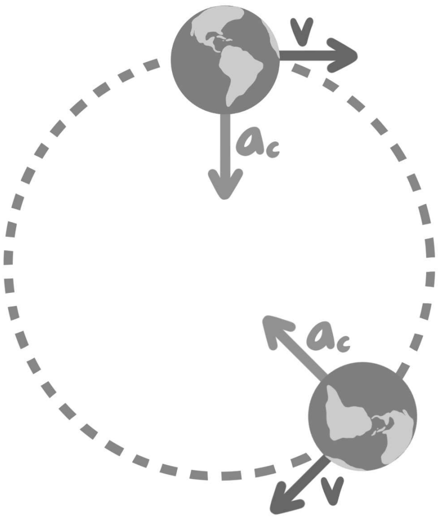
Centripetal acceleration

As an object changes direction its velocity changes (even if the size of the velocity does not!)

This is called centripetal acceleration.

A force or combination of forces providing centripetal acceleration are called centripetal forces.



Centripetal acceleration points to the centre.

Velocity points tangential (90° to acceleration).

$$F_c = ma_c = \frac{mv^2}{r}$$

Mass (kg)

Velocity (ms^{-1})

Radius (m)

Centripetal force (N)

Centripetal acceleration (ms^{-2})

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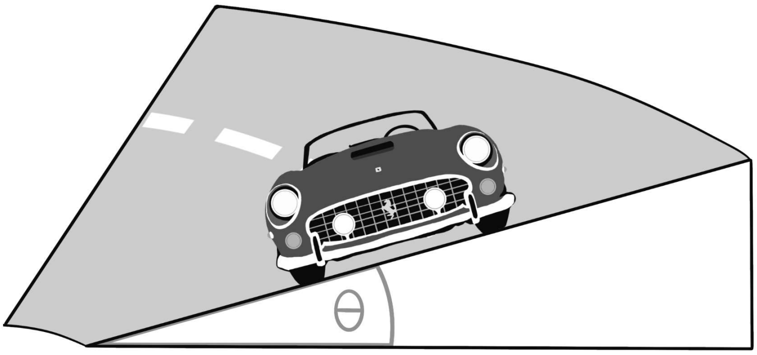
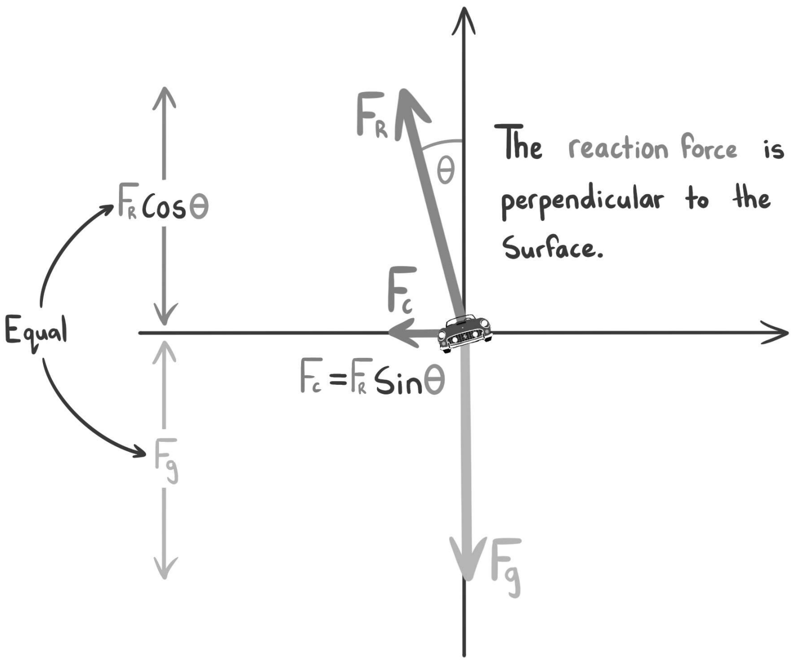
Banked corner



The centripetal force is provided by the unbalanced horizontal force. In this case it is the horizontal component of the reaction force $F_R \sin\theta$.



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Newton's law of gravity

All objects with mass attract each other.

Two masses m_1 and m_2, a distance r apart, will attract each other with a force...

Mass 1 (kg) Mass 2 (kg)

Gravitational force (N)

$$F_g = G\frac{m_1 m_2}{r^2}$$

Universal gravitational Constant ($6.67 \times 10^{-11}\ Nm^2kg^{-2}$)

Separation (m)

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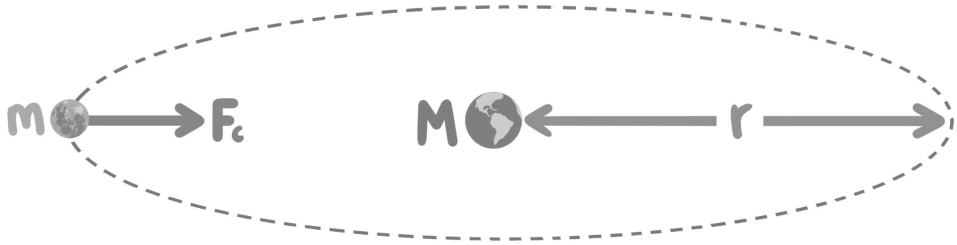
Orbital motion

Orbital motion is circular motion where the centripetal force is provided by the gravitational force.



$$F_c = F_g$$

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

$$v^2 = \frac{GM}{r}$$

Velocity (ms^{-1})

$$v = \sqrt{\frac{GM}{r}}$$

Mass of attracting body (kg)

Separation (m)

Universal gravitational Constant ($6.67 \times 10^{-11}\ Nm^2kg^{-2}$)

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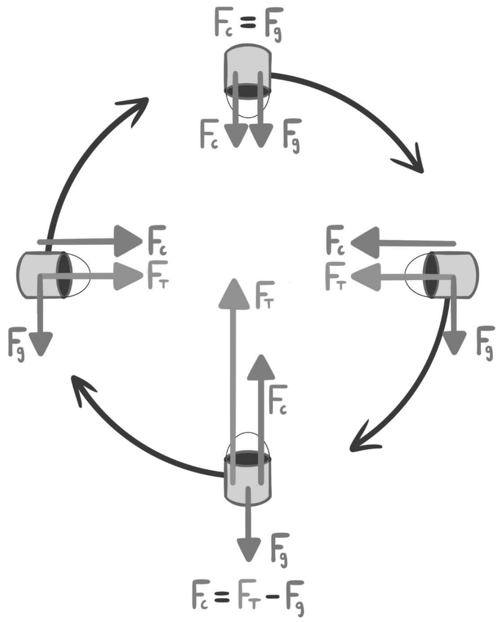
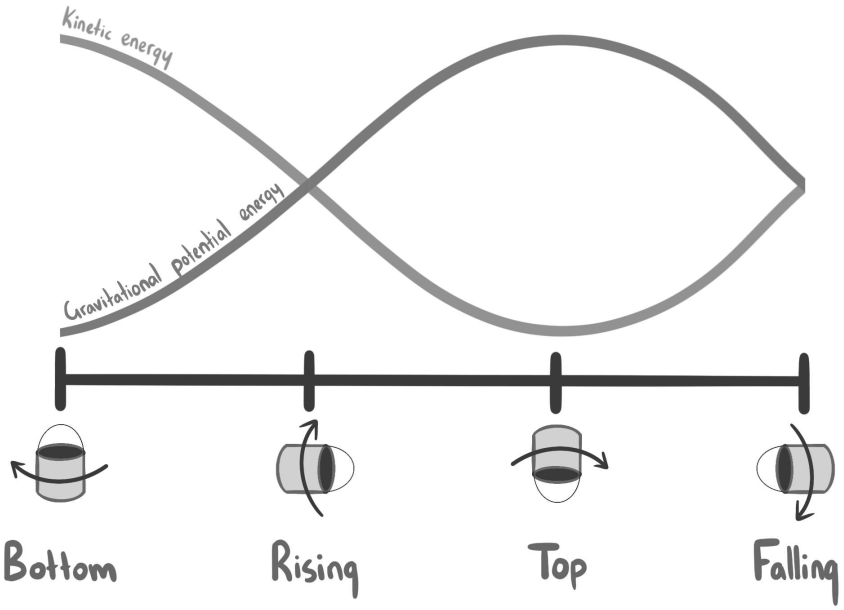
Vertical Circular motion

A bucket is swung in a vertical circle at the minimum Velocity required.

The centripetal force is provided by the Tension and/or gravity force at different times.





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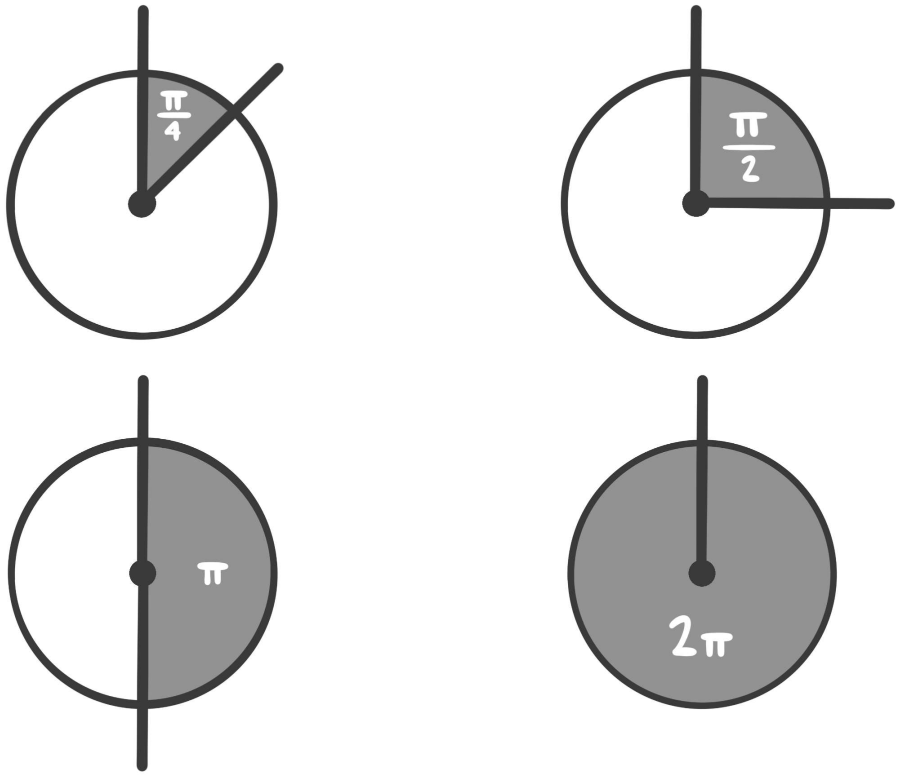
Measuring angle

Angle can be measured in either degrees (°) or radians (rad).

$$1\,\text{rad} = 57.3^\circ$$

We may also express an angle as a multiple of pi (π).

$$2\pi = 1 \text{ revolution} = 360^\circ$$



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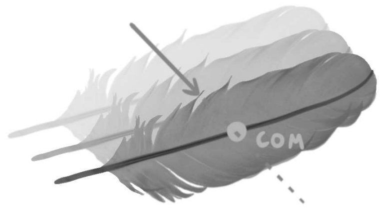
Translational vs Rotational

Pure translational motion occurs when the force points through the centre of mass.



Pure rotational motion occurs when the net force is zero but the net torque is not.

Otherwise a mix of translational and rotational motion will occur.

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Rotational motion

θ Angular displacement

Change in angle.

Measured in radians (rad).

ω Angular velocity

Rate of θ change.

Measured in radians per second ($rad\ s^{-1}$).

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Δt ← Duration (s)

α Angular acceleration

Rate of ω change.

Measured in radians per second, per second ($rad\ s^{-2}$).

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

Δt ← Duration (s)

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Equations of rotation

As with translational motion we can use kinematic equations to describe rotational motion with constant angular acceleration.

Initial angular velocity
ω_i

Final angular velocity
ω_f

Time
t

Angular displacement
θ

Angular acceleration
α

$$\omega_f = \omega_i + \alpha t$$ (Missing θ)

$$\theta = \omega_i t + \tfrac{1}{2}\alpha t^2$$ (Missing ω_f)

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$ (Missing t)

$$\theta = \frac{\omega_i + \omega_f}{2} t$$ (Missing α)

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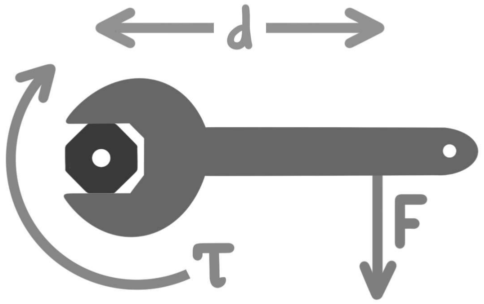
Torque

A force is an influence that acts to accelerate an object.

A torque is an influence that acts to angularly accelerate an object.

It has units of Newton metres (Nm).



Force (N)

$$\tau = Fd$$

Torque (Nm)

Distance from pivot (m)

A force on a mass causes acceleration described by $F = ma$.

A torque on a rotational inertia causes rotational acceleration described by...

Angular acceleration (rad s^{-2})

$$\tau = I\alpha$$

Torque (Nm)

Rotational inertia (kgm^2)

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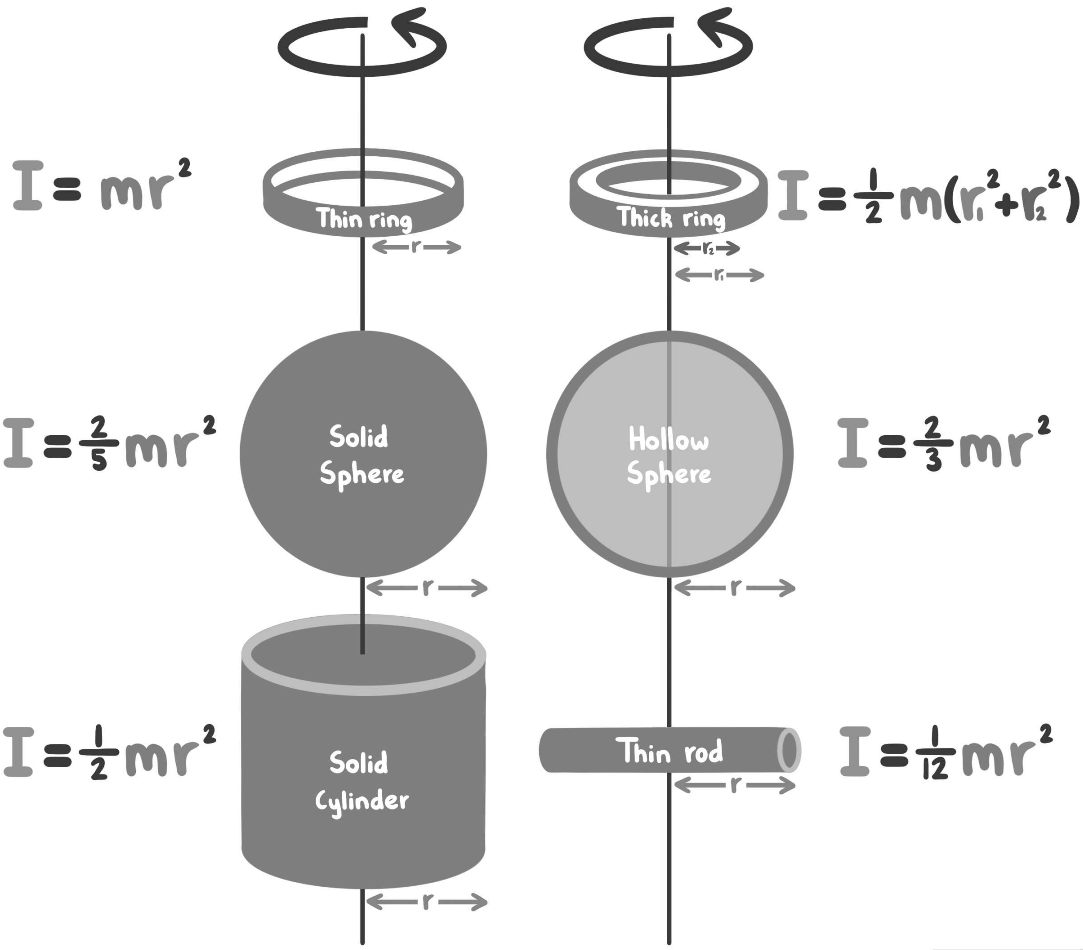
Rotational inertia

Mass is resistance to acceleration.

Rotational inertia is resistance to angular acceleration.

Rotational inertia has units of kgm^2.



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Rotational kinetic energy

The kinetic energy of a rotating object is described by...

$$E_{K(ROT)} = \frac{1}{2} I \omega^2$$

Rotational inertia (kgm^2)

Angular velocity ($rad\,s^{-1}$)

Rotational kinetic energy (J)

The **total kinetic energy** of an object is the sum of the **linear** and **rotational** kinetic energies.

$$E_{K(ToT)} = E_{K(LIN)} + E_{K(ROT)}$$

Total kinetic energy (J) Linear kinetic energy (J) Rotational kinetic energy (J)

(Bullet block vid)

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Angular momentum

Linear momentum describes the amount of Linear motion an object has, taking into account its mass (linear inertia).

Mass (kg)

Momentum ($kgms^{-1}$) → $$p = mv$$ ← Velocity (ms^{-1})

Angular momentum does this in a rotational context...

Rotational inertia (kgm^2)

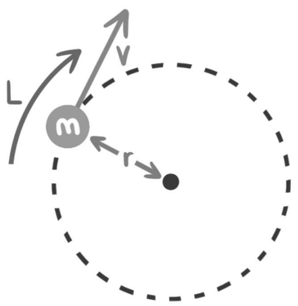
Angular Momentum (kgm^2s^{-1}) → $$L = I\omega$$ ← Angular velocity ($rads^{-1}$)

We can relate angular momentum and linear momentum with each other...

Mass (kg)

Angular Momentum (kgm^2s^{-1}) → $$L = mvr$$

Velocity (ms^{-1}) Radius (m)



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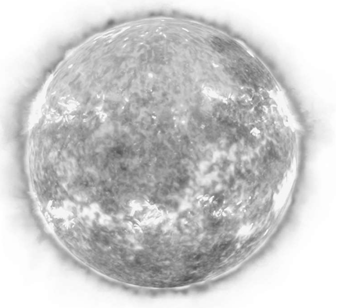
Angular momentum Conservation

As with linear momentum, angular momentum is always conserved (provided there are no external torques).

Consider a main sequence star collapsing to form a neutron star.



~1 rotation every 30 days

~700 rotation every second

$$L = I\omega$$

Angular momentum is conserved

$$L = I\omega$$

Angular velocity increases

Rotational inertia decreases

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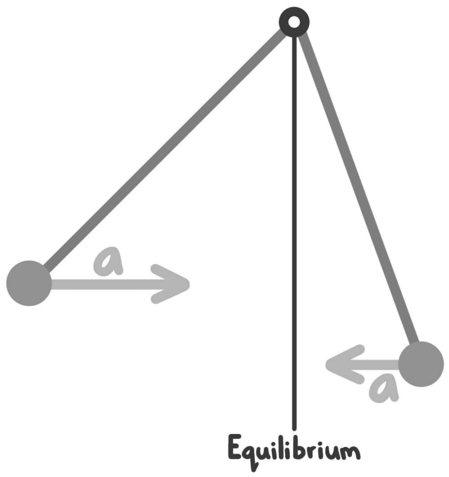
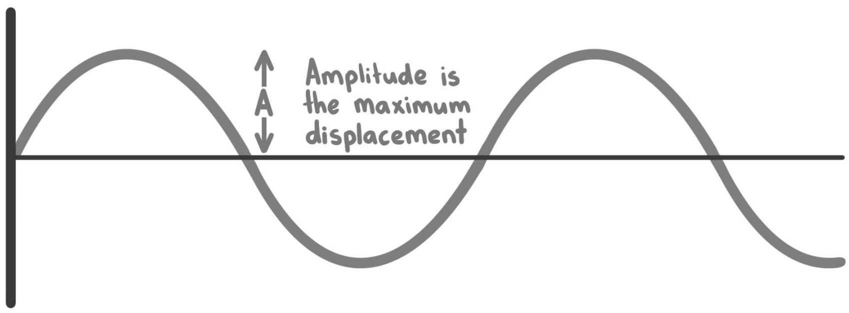
Simple harmonic motion

By definition system in simple harmonic motion...

- Acceleration towards equilibrium.
- Accelerates proportional to its displacement.





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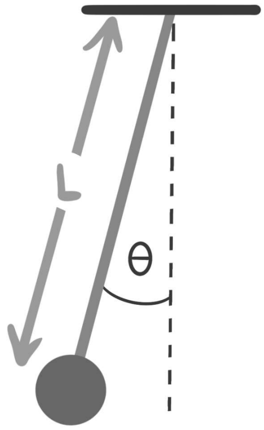
The simple pendulum



Period (s)

Length (m)

$$T = 2\pi\sqrt{\frac{L}{g}}$$

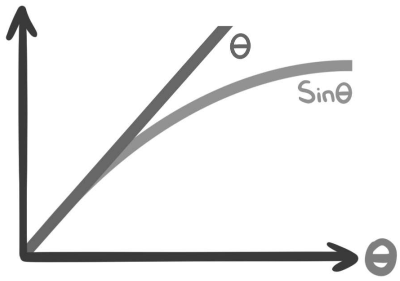
Gravitational acceleration (ms^{-2})

This equation assumes the small angle approximation. This requires that the angle must be small.

The equation specifically assumes that...

$$\sin\theta \approx \theta$$

This is only reasonable when the angle is small.



At ≈10° the error exceeds 1% and starts to affect values expressed to 3sf, which is standard at NCEA Level 3.

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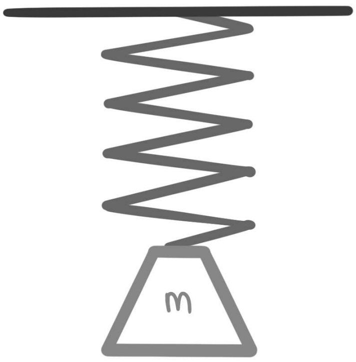
Mass on a Spring



A mass on a spring displaced from equilibrium will oscillate with a period described by...

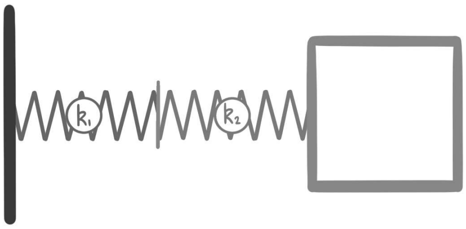
$$T = 2\pi\sqrt{\frac{m}{k}}$$

Period (s)

Mass (kg)

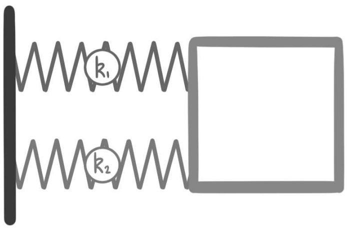
Stiffness (Nm^{-1})

* Springs in series



$$\frac{1}{k_{Total}} = \frac{1}{k_1} + \frac{1}{k_2}$$

* Springs in parallel



$$k_{Total} = k_1 + k_2$$

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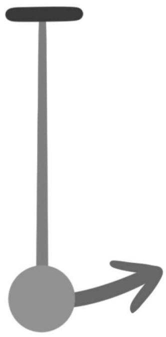
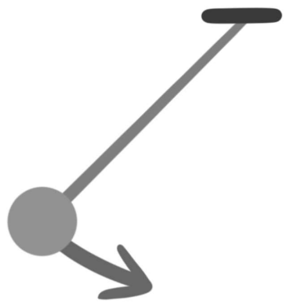
SHM Equations

A system in simple harmonic motion follows a sinusoidal relationship.

Note that $\theta = \omega t$.





✱ Begins at $y=0$

Amplitude (m)

Displacement (m)

$y = A\sin\omega t$

Angular frequency (rad s^{-1}) Time (s)

Velocity (ms^{-1})

$v = A\omega\cos\omega t$

Acceleration (ms^{-2})

$a = -A\omega^2\sin\omega t$

✱ Begins at $y = A$

Displacement (m)

$y = A\cos\omega t$

Velocity (ms^{-1})

$v = -A\omega\sin\omega t$

Acceleration (ms^{-2})

$a = -A\omega^2\cos\omega t$

Note that $v_{max} = A\omega$ and $a_{max} = A\omega^2$

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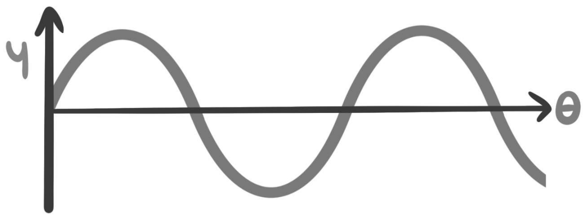
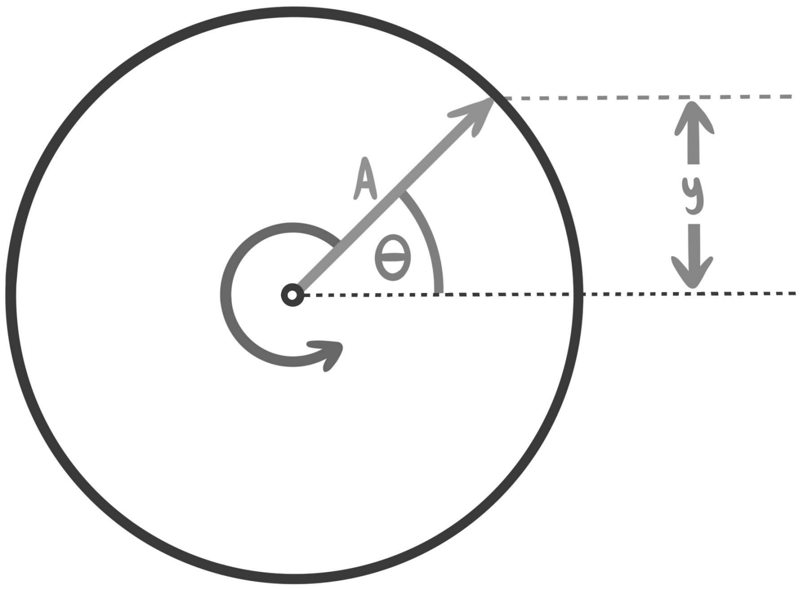
SHM and Circular motion

Until now we've represented SHM as a displacement-time graph however, we can also represent SHM as an analog of Circular motion called phasors.





(Animation)

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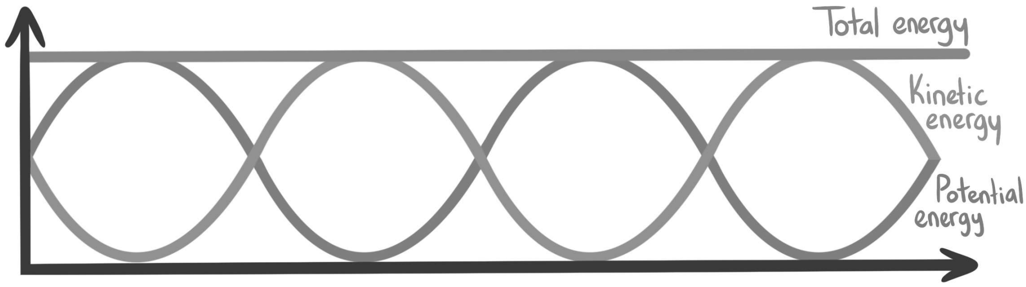
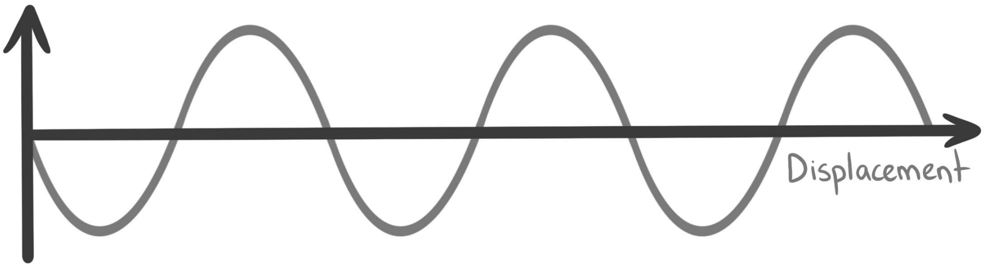
Energy of SHM

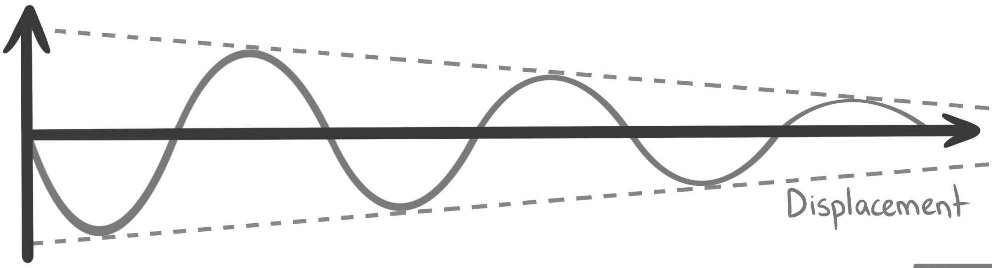
The energy of a system in SHM is exchanged between kinetic and potential.



The above example assumes that the total energy remains constant.



In many systems however, energy is gradually lost. We call this damping.



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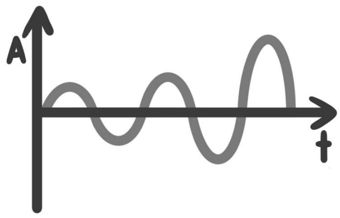
Resonance

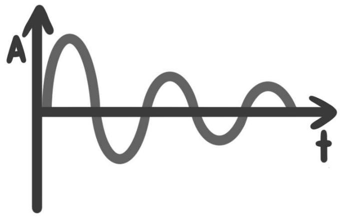
Adding energy to an oscillating system is called driving.

Every oscillating system has a natural frequency.

Inputs of energy at this frequency will constructively interfere with the existing energy of the system.



Inputs of energy above or below this frequency will destructively interfere with the existing energy of the system.



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